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EFFECTS OF ECCENTRICITY OF BOATTAIL AND MASS IN 155MM HOWITZER H.E. SHELL MK I

by

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EFFECTS OF ECCENTRICITY OF BOATTAIL AND MASS IN 155MM. HOWITZER H.E. SHELL MK.I

Abstract

The effects upon range, deflection and dispersion resulting from abnormal dimensions or mass distribution in shell must be kept small if accuracy of fire is to be ensured. The action of general abnormalities of shell is traced through four different regimes of the flight. Measurements of four particular abnormalities of shell are discussed to some extent. Theoretical effects are computed for some abnormalities of shell and estimated for others. Experimental determination of corresponding effects is undertaken from two samples of abnormal shell. These results are compared with similar experimental determinations made by R. H. Kent for 155mm Gun H.E. Shell Mk.III. The methods suggested in this report should have application to the determination of allowable manufacturing tolerances for shell.

CONTENTS

A. Theoretical and Numerical Basis for Computing the Effects of Abnormal Dimensions of Shell on Range, Deflection and Dispersion

I. List of Basic Symbols and Definitions

In order to provide for a mathematical discussion of the phenomena governing the effects of eccentricity of shell, certain symbols and definitions will be introduced. These symbols will be ordered according to subject matter and their meanings will be discussed further in the text.[1]

L_i: a typical physical characteristic of a projectile used without subscript if it is unnecessary to specify the particular characteristic discussed;

λ : population mean value of L for normal shell;
λ': population mean value of L for abnormal shell;
$\underline{L}$: mean value of L for a sample of normal shell;
L': mean value of L for a sample of abnormal shell;
L_D: design value of L for a particular type of shell;
$\varepsilon_L = L - \lambda$: generalized departure from population mean value of L;
σ_L^2 : population variance in L for normal shell;
$\sigma_L^{2\prime}$: population variance in L for abnormal shell;
s_L^2: sample variance in L for normal shell;
$s_L^{2\prime}$: sample variance in L for abnormal shell;
$\widehat{s_L^2}$: optimum estimate of population variance in L for normal shell;
$\widehat{s_L^2}'$: optimum estimate of population variance in L for abnormal shell;
t_L: tolerance in ε_L.

Particular instances of L referred to in the present report are:

β : eccentricity of boattail, that is, one half the difference of maximum and minimum indicator readings in inches taken at one half inch from the shell base;
r: eccentricity of center of mass, that is, the distance from the center of mass of the shell to its longitudinal axis of figure;
b: band position, that is, the distance of the rear of the band from the shell base;
d: diameter of the bourrelet;

Each of the above possess population means, sample means, design values, population variances, sample variances and tolerances.

[1] The notation here employed is largely that of H. P. Hitchcock, who gives many formulae and units for use in the treatment of exterior ballistics in Ballistic Research Laboratory Report 111: "Aerodynamic Formulae and Nomenclature."

Some discussion of quantities algebraically related to the above will be given. These are:

c: eccentricity of cavity, that is, the distance between the longitudinal axis of the shell surface and the longitudinal axis of the cavity.

δm: eccentricity of mass, that is, the mass necessary to add to the light side of the shell at distance R from the axis of figure in order to make the longitudinal axis of mass distribution coincide with the longitudinal axis of figure;

F: flat length, that is, the distance between the boattail and the band;

Certain other characteristics of shell involved in the discussion are:

m: mass of the shell;

A: moment of inertia about the nearly longitudinal principal axis of inertia. In this report it will be presumed that the longitudinal principal axis is parallel to the axis of figure;

B: least moment of inertia of the shell about a principal transverse axis through the center of mass;

k_B: radius of gyration of the shell about a transverse axis through the center of mass;

l_g: distance from the center of mass to the center of the band;

l_1: distance from the point of impact of the bourrelet with the land top to the diametral plane through the band;

l_i: the distance from the center of mass to the diametral plane through the point on the shell where the impact of the bourrelet with the land top takes place;

d_i: diameter of the bourrelet at the point of impact with the land top;

g: distance from the base of the shell to the center of the mass;

x: distance of a point on the shell axis measured from the base of the shell;

V_c: total volume of the shell cavity with fuze in place;

T: thickness of the shell wall;

R: radius of the shell between the band and the bourrelet;

e: coefficient of restitution of the bourrelet;

$C = \frac{m}{id^2}$ = ballistic coefficient of the shell;

C_L: drift coefficient of the shell;

i: coefficient of form of the shell;

K_D: drag coefficient for zero yaw;

K_{D_δ}: yaw drag coefficient, that is coefficient of augmentation of drag due to yaw;

K_L: lift or cross-wind force coefficient;

K_M: overturning moment coefficient;

K_H: damping coefficient;

$S_o = \dfrac{\pi^2 A^2}{\rho_\omega n^2 d^5 K_M}$: initial stability factor of the shell;

$S = S_o\left(\dfrac{v_o}{v}\right)^2$: approximate general stability factor of the shell;

Quantities associated with the motion of the shell include:

θ: angle between the axis of the shell and the axis of the bore due to clearance inside the bore. It is presumed that the band centers the shell at the band;

θ_o: value of θ after ramming;

θ : value of θ at the instant the band disengages;

θ_M: maximum value of θ;

ψ : orientation of the center of mass with reference to the slant plane through the gun bore. This angle is measured clockwise as seen from the chamber;

ψ_o: value of ψ after ramming;

ψ : value of ψ at the instant the band disengages;

z_b: travel of the band from seating position;

Z_b: travel of the band from the position of the band at seating to the muzzle;

Z_g: travel of the center of mass from the position at seating to the muzzle;

$\mathfrak{E}_c$: yaw of the shell at the instant just after disengagement of the band due to clearance inside the bore: angle between the axis of the shell and the tangent to the trajectory on emergence of the band;

$\varphi_{\mathfrak{E}c}$: orientation of the nose of the shell with reference to the vertical plane through the tangent to the trajectory at the disengagement of the band;

$\mathcal{E}_c$: initial yaw of the shell: yaw due to clearance obtained by backward extrapolation from instant of disengagement of the band to instant of passage of the muzzle by the center of mass;

$\varphi_{\mathcal{E}c}$: orientation of the nose of the shell with reference to the vertical plane through the tangent to the trajectory at the instant of emergence of the center of mass;

J : jump due to yaw due to clearance;

φ_J: orientation of J with reference to the vertical plane through the gun bore;

$\gamma = \varphi_J - \frac{\pi}{2}$;

γ: the time interval from the instant the explosion starts to the instant the band passes the muzzle;

j: jump in the vertical plane due to angular motion of the gun bore;

i_1: side jump in the slant plane due to angular motion of the gun bore;

δ: yaw of the shell at any time;

α: amplitude of δ;

α_o: initial amplitude of the yaw;

v: velocity of the center of mass of the shell;

v_o: muzzle velocity;

G: resistance function;

$H=e^{+ay}$: standard ratio of the density of the air at any altitude to that at sea-level;

$E_1 = \frac{GH}{C}$;

x: range coordinate of the center of mass of the shell at any time, t;

y: altitude of the center of mass of the shell above sea-level at any time, t;

z: deflection coordinate of the center of mass of the shell at any time, t;

E: angle of elevation of the gun bore above sea-level;

ϕ: angle of departure of the center of mass of the shell above sea-level;

$I=i_1 + J \sin\varphi_J$: total side jump of the center of mass of the shell in the slant plane;

N_o: spin of the shell at the muzzle;

$$J_1 = \frac{\mathcal{E}c}{\sinh j_1};$$

$$j_1 = \tanh^{-1}\left[\frac{\left(\frac{S_o - 1}{S_o}\right)^{1/2}}{\left(\frac{2B}{A} - 1\right)}\right];$$

$$f = \frac{\rho d^4 v K_H}{B};$$

$$k = \frac{\rho d^2 v K_L}{m};$$

$$h_1 = \int_0^t (f+k)\, dt;$$

$$h_2 = \frac{1}{2}\int_0^t (f-k)\left(\frac{S}{S-1}\right)^{1/2} dt;$$

X: range of the shell;
Z: deflection of the shell;
ζ: drift of the shell;
h: elevation of the trunnions of the gun above the point of impact;
$\Delta_{m_c} X$: mass effect on range through mass effect on ballistic coefficient;
$\Delta_{m_{v_o}} X$: mass effect on range through mass effect on muzzle velocity;
$\Delta_{v_o} X$: velocity effect on range due to non-zone value of the muzzle velocity;
$\Delta_p X$: effect of temperature of powder on range;
$\Delta_{w_x} X$: effect of ballistic range wind on range;
$\Delta_H X$: effect of ballistic density on range;
$\Delta_\tau X$: effect of ballistic temperature on range;
$\Delta_h X$: effect of depth of point of impact on range;
$\Delta_{w_z} Z$: effect of ballistic cross wind on deflection;
$\Delta_k Z$: effect of cant on deflection;
$\Delta_L X$: effect of the departure ε_L upon range;
$\Delta_L Z$: effect of the departure ε_L upon deflection;
σ_X^2 : population variance in range;
σ^2_Z : population variance in deflection;

Certain characteristics of the howitzer also require definition, namely:

d_H : diameter of the bore between land tops and
n : number of calibers length per full turn of rifling.

The final results to be found are coefficients which, when multiplied by the abnormality, ε_L, furnish the effects of ε_L on the elements range and deflection at a range of 9600 yards with muzzle velocity 1478 feet per second with the 155mm. Howitzer H.E. Shell Mk. I. These are:

$$c_\beta = \frac{dX}{d\beta}$$

$$\left\{\begin{aligned} c_{\delta m} &= \frac{dX}{d\delta m} \\ c_c &= \frac{dX}{dc} \end{aligned}\right.$$

$$c_b = \frac{dX}{db}$$

$$c_d = \frac{dX}{dd}$$

$$k_\beta = \frac{dZ}{d\beta}$$

$$\left\{\begin{aligned} k_{\delta m} &= \frac{dZ}{d\delta m} \\ k_c &= \frac{dZ}{dc} \end{aligned}\right.$$

$$k_b = \frac{dZ}{db}$$

$$k_d = \frac{dZ}{dd} .$$

II. Dynamical Consequences of Asymmetry of Boattail and Mass Distribution in Shell.

1. General

Projectiles with very slightly different shape parameters, surface roughness or mass distribution frequently have very different aerodynamic coefficients. Notably differing aerodynamic coefficients result in correspondingly differing flight characteristics. The initial conditions, the form of the equations of motion and the aerodynamic coefficients occurring therein are, in the case of shell which are asymmetric in shape, surface roughness or mass distribution, all radically different from the corresponding case with normal shell. The effects upon range, deflection and dispersion resulting from abnormal dimensions or mass distribution must be kept small if accuracy of fire is to be ensured.

The design of shell is based in part on the results of experiment and aerodynamic theory. The manufacturing tolerances upon the design should be governed in part by the magnitude of the permissible effects upon range, deflection and dispersion resulting from variation of the shell within these tolerances. Some experimental determinations of the magnitude of the effects on range and dispersion due to various abnormalities of shell have been made at the proving ground.[1]

2. Phases of Action of Abnormalities in Shell

The action of abnormalities of shell is different in different regimes of the flight and four such regimes will be distinguished:

[1] Particular reference will be made in the present report to a report of R. H. Kent: "Memorandum Report on Tolerance Tests of 155mm. Projectiles O.P. No. 4084." The latter report is a comprehensive examination of a carefully designed experimental program on Tolerance Tests of 155mm. Gun H.E. Shell Mark III. Valuable material containing a treatment of some dynamic characteristics of asymmetric shell is included in other reports, notably: H. P. Hitchcock: "Unbalance of 3" C.S. Shell, Model 1915;" H. P. Hitchcock: "Eccentricity of 155mm. Shell."

(1) the regime inside the gun;

(2) the regime from the emergence of the band from the muzzle to the first maximum yaw;

(3) the regime from the first maximum yaw to a distance from the gun sufficient to render the yaw dominantly precessional;

(4) the regime from the region where the yaw has become dominantly precessional to the point of impact.

3. Effect of Generalized Abnormalities of Shell

Let a typical physical characteristic, that is, a dimension, mass distribution or degree of surface roughness be denoted by L_i. For the present but one such characteristic will be considered at a time and its varying value will be denoted simply by L. For an indefinitely great number of normal shell, L will have a population mean value λ. The construction of firing tables should be based upon the range firing of shell which have, for each sample mean $\bar{L}$, a value very nearly equal to the population mean value λ; this will assure the using service of correct mean points of impact in the long run. The design value of the shell will be denoted by $L_{\mathfrak{D}}$. In general $L_{\mathfrak{D}}$ will differ from λ by a small quantity, $L_{\mathfrak{D}}-\lambda$, which quantity arises from the minute systematic error of manufacture.[1] Then a truly normal shell will be presumed to have very nearly the population mean dimensions λ_i. The firing table range X and the deflection Z will be supposed to correspond to the population mean values of $\bar{X}$ and $\bar{Z}$ attained by an infinite number of shell with mean dimensions λ_i to a sufficient degree of accuracy for present purposes. Let ε_L denote a generalized departure from the population mean value of L. Then:

$$\varepsilon_L = L - \lambda . \qquad (1)$$

[1] In at least certain cases $L_{\mathfrak{D}} - \lambda$ can never vanish. In considering the eccentricity of cavity, c, it is clear that the design value $c_{\mathfrak{D}}$ is zero, whereas the mean value of c for shell made under similar circumstances will approach a small positive number c_1 within limits which become indefinitely close as the number of shell measured becomes indefinitely great.

It is clear that the population mean value of ε_L for an indefinitely great number of normal rounds is zero. On the other hand, the actual individual shell, by reason of the unavoidable imperfections of material and workmanship, possess values of ε_L which vary in a random manner. For an infinite sequence of such shell, ordered by magnitude of ε_L, ε_L may be presumed to take on a range of values in a continuous manner. Then ε_L, the generalized abnormality, can be considered a random variable possessing a continuous distribution function, that is, a variable which is capable of assuming any value within a given range with a definite probability. It should be noted that while the population mean value of ε_L is zero, the population mean square of ε_L for an indefinitely great number of shell will not be zero. This mean square of ε_L is described as the population variance of L. Then:

$$\left.\begin{aligned} \bar{\varepsilon}_L &= 0 \\ \sigma_L^2 &= \lim_{n\to\infty} \frac{\sum_n \varepsilon_L^2}{n} . \end{aligned}\right\} \quad (2)$$

For any finite size of sample, the variance obtained by direct calculation is:

$$s_L^2 = \frac{\sum_n (L - \bar{L})^2}{n} . \qquad (3)$$

In general the population variance must be estimated from variance of the sample. The mean value of the ratio of the variance of the sample to the corresponding population parameter is

$$\frac{n - 1}{n} .$$

The "optimum" estimate of σ_L^2 is obtained by [1]

$$\hat{s}_L^2 = \frac{n}{n - 1} s_L^2 . \qquad (4)$$

[1] The term "optimum estimate" is used by R. A. Fisher to denote the estimate obtained by the method of maximum likelihood. This formula is given in a different notation by W. A. Shewhart: "Economic Control of Quality of Manufactured Product," page 188. The quantity $\hat{s}^2$ is Shewhart's σ^2 and s^2 is Shewhart's $\bar{\sigma}^2$. The result is well-known.

The firing table probable errors in range and deflection for an individual round should be determined as the probable error from shell which have for each sample variance s_L^2, an estimate $\hat{s}_L^2$ derivable therefrom which agrees very nearly with the population variance, σ_L^2. In this way the using service is assured a probability correct in the long run, of hitting with the actual shell possessed by the batteries. Accordingly, it will be assumed in the present report that the firing table probable errors approximate the population probable errors of individual shell which, for the tabular points have a sample variance that is rendered close to σ_L^2 upon correction by equation (4). This is equivalent to the assumption that an indefinitely great number of normal shell have variances in range and deflection equal to σ_X^2 and σ_Z^2 respectively. A great number of shell, each having a single constant value of ε_L, will then have mean coordinates given by:

$$\left.\begin{aligned} X &= \overline{X} + \Delta_L X \\ Z &= \overline{Z} + \Delta_L Z \end{aligned}\right\} (5)$$

where $\Delta_L X$ and $\Delta_L Z$ are the mean effects of ε_L. It will be supposed that the values of ε_{L_i} are small in the sense that their mean effects may be assumed proportional to ε_{L_i}. Then, as a consequence of the latter assumption, the mean effects of ε_{L_i} are all independent, the effect of the sum of ε_{L_i} being the same as the sum of the effects of each ε_{L_i} taken separately.

Then:

$$\left.\begin{aligned} \Delta_L X &= \frac{dX}{dL}\,\varepsilon_L = c_L \varepsilon_L \\ \Delta_L Z &= \frac{dZ}{dL}\,\varepsilon_L = k_L \varepsilon_L . \end{aligned}\right\} (6)$$

With regard to dispersion, let it be supposed that there exists a variance in range and a variance in deflection which will always occur because of irremovable conditions unassociated with design. These variances are those which would be present even if ε_L were zero. Then if the manufacture of shell were carried out in perfectly exact agreement with the population mean values λ_i, the variances in range and deflection would be given by the quantities here denoted by $\sigma^2_{X_o}$ and $\sigma^2_{Z_o}$. These latter, the variances of perfect shell, are evidently independent of the variances due to the action of the abnormalities ε_L. The variances of normal shell are obtained by the law for compounding independent errors:

$$\left.\begin{aligned} \sigma^2_X &= \sigma^2_{X_o} + \sigma^2_{\Delta_L X} = \sigma^2_{X_o} + \left(\frac{dX}{dL}\right)^2 \sigma^2_L \\ \sigma^2_Z &= \sigma^2_{Z_o} + \sigma^2_{\Delta_L Z} = \sigma^2_{Z_o} + \left(\frac{dZ}{dL}\right)^2 \sigma^2_L . \end{aligned}\right\} \quad (7)$$

In the case of significantly abnormal shell, a group may exist which have a greater variance than σ^2_L. Let this abnormal population variance of L be denoted by $\sigma^{2\prime}_L$, and let the corresponding variances in range and deflection be denoted by $\sigma^{2\prime}_X$ and $\sigma^{2\prime}_Z$. Evidently:

$$\left.\begin{aligned} \sigma^{2\prime}_X &= \sigma^2_X + \Delta_L \sigma^2_X = \sigma^2_X + \left(\frac{dX}{dL}\right)^2 \Delta \sigma^2_L = \sigma^2_X + \left(\frac{dX}{dL}\right)^2 (\sigma^{2\prime}_L - \sigma^2_L) \\ \sigma^{2\prime}_Z &= \sigma^2_Z + \Delta_L \sigma^2_Z = \sigma^2_Z + \left(\frac{dZ}{dL}\right)^2 \Delta \sigma^2_L = \sigma^2_Z + \left(\frac{dZ}{dL}\right)^2 (\sigma^{2\prime}_L - \sigma^2_L) . \end{aligned}\right\} \quad (8)$$

Then the effects upon the variances in range and deflection which result from the augmentation of the variance in L are $\Delta_L \sigma^2_X$ and $\Delta_L \sigma^2_Z$. These effects are:

$$\left.\begin{aligned} \Delta_L \sigma^2_X &= \left(\frac{dX}{dL}\right)^2 \Delta\sigma^2_L = c^2_L \Delta\sigma^2_L \\ \Delta_L \sigma^2_Z &= \left(\frac{dZ}{dL}\right)^2 \Delta\sigma^2_L = k^2_L \Delta\sigma^2_L . \end{aligned}\right\} \quad (9)$$

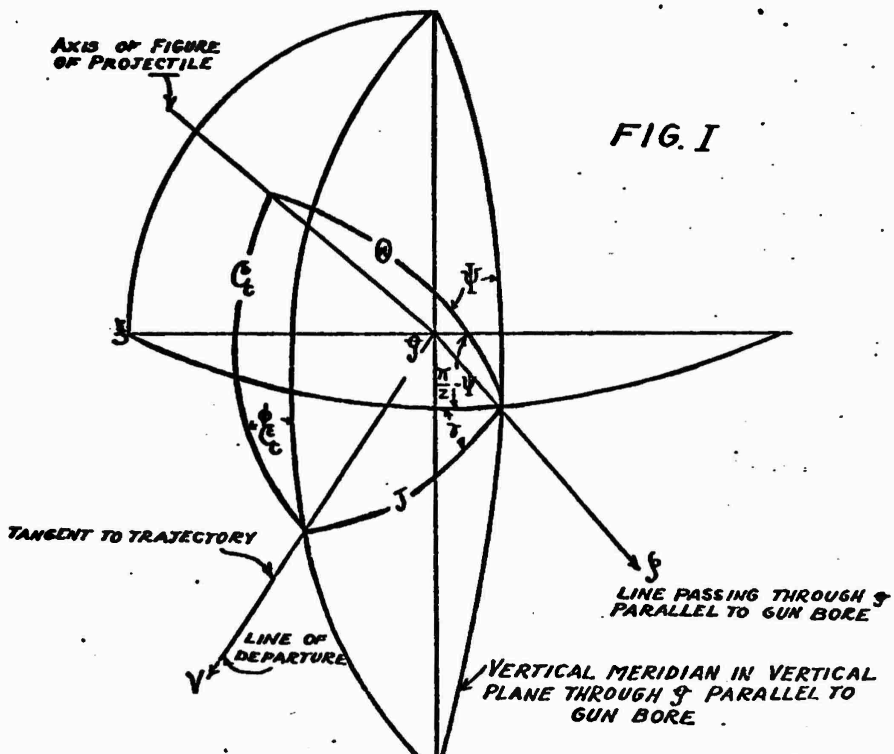
If more than one departure from normal population characteristic is present, both sides of (6) and (9) become the sum of terms of which these single terms are typical. Then the total effects on range and deflection and the variances of range and deflection are the sums of the partial effects.

The dynamical consequences of ε_{L_1} are the four regimes may all be discussed qualitatively and are frequently capable of complete theoretical treatment and a priori calculation. Some discussion of the dynamical consequences of the abnormalities will be given now. In general reliable quantitative determination of the effects of most abnormalities can only be obtained by experiments such as those described later.

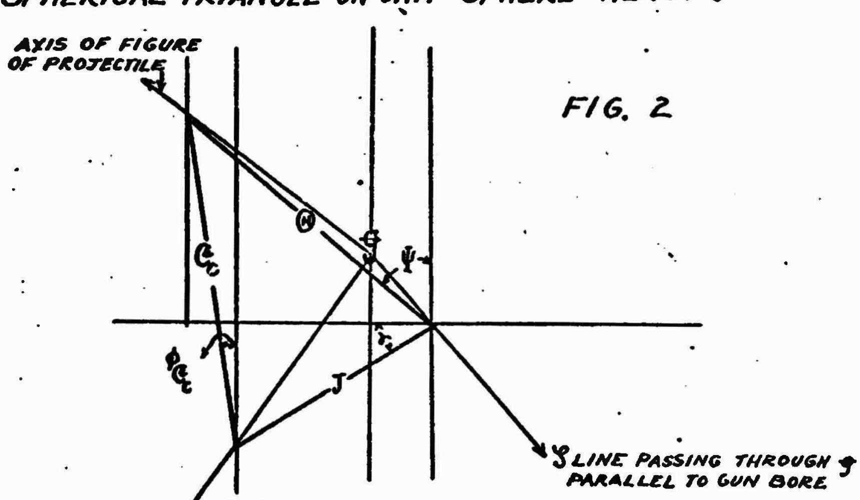
In the first regime, the action of ε_L will be such as to modify the yaw of the shell, the orientation of the yaw and the jump on emergence of the center of mass due to the yaw due to clearance inside the bore of the gun. Free employment of the definitions and symbols given in Section I of this report will be made. It has been shown[1] that the yaw on emergence of the band from the muzzle due to clearance, δ_c, and the orientation of that yaw at the same instant, $\varphi\delta_c$, may be calculated precisely by:

$$\left.\begin{aligned} \delta_c &= \sqrt{\theta^2 + J^2 + 2\theta J\sin(\gamma - \psi)} \\ \varphi\delta_c &= \gamma - \text{arc sin}\left(\frac{\delta_c^2 + J^2 - \theta^2}{2\delta_c J}\right). \end{aligned}\right\} \qquad (10)$$

[1] F. V. Reno: "The Motion of the Axis of a Spinning Shell Inside the Bore of the Gun", Ballistic Research Laboratory Report No. 320.



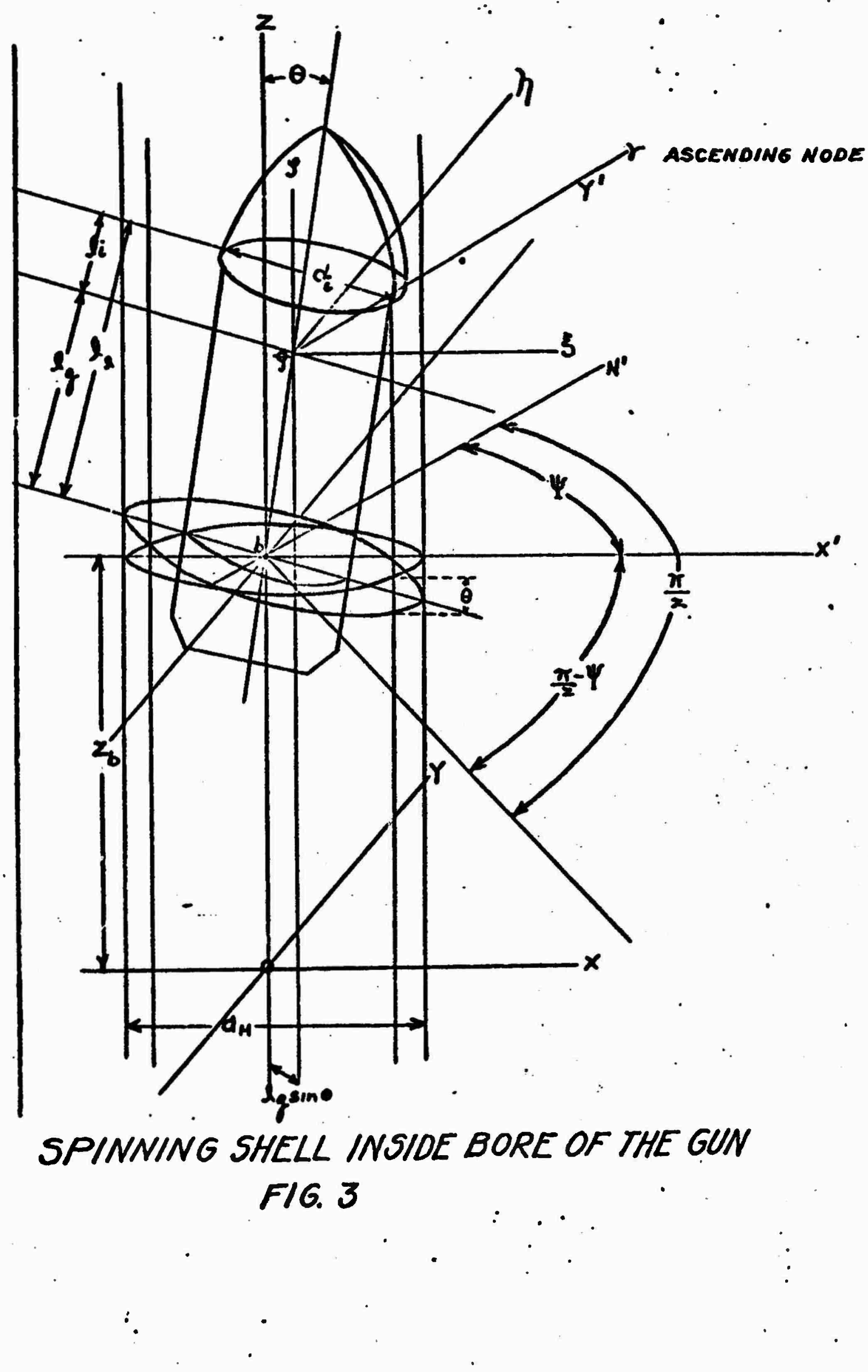
SPHERICAL TRIANGLE ON UNIT SPHERE ABOUT $\mathfrak{g}$



PLANE TRIANGLE TANGENT TO UNIT SPHERE

The notation followed in this report differs slightly from that in the former report cited. It is desirable to indicate that θ is the angle between the axis of the shell and the axis of the bore due to clearance inside the bore on the supposition that the band centers the shell at the band. The angle ψ is the orientation of the center of mass with reference to the slant plane through the gun-bore. This angle is measured clockwise as seen from the chamber. The angles θ and Ψ are the values assumed by θ and ψ at the instant the band disengages. In general, J, the jump on emergence due to yaw due to clearance and γ, the orientation of the jump, J, with reference to the slant plane through the bore, are constants for a single round fired. On the other hand θ, $\dot{\theta}$, ψ and $\dot{\psi}$ vary with the time as counted from the instant of initiation of the explosion. In the source just cited it is shown that θ is determined by solution of the differential equation:

$$\ddot{\theta} = \frac{\left[(B + ml_g^{\,2} - A)\left(\frac{2\pi\dot{z}_b}{nd_H}\right)^2 + ml_g\ddot{z}_b\right]}{(B + ml_g^{\,2})}\,\theta \qquad (11)$$



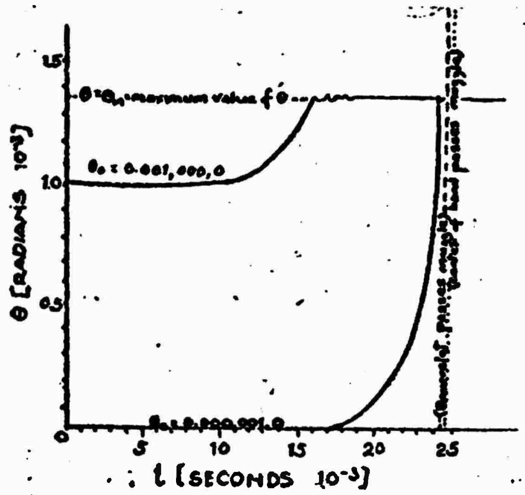
SPINNING SHELL INSIDE BORE OF THE GUN

FIG. 3

with the initial conditions $\theta(0) = \theta_o$ and $\dot{\theta}(0) = 0$. The subscript zero denotes a value after ramming, that is the value assumed by a variable quantity at the initiation of the explosion. The solution of this differential equation governs the motion except at the instants when the bourrelet strikes the land top. At the instant when the bourrelet strikes the land top, there is an impulsive change in the motion, which results in a requirement that the solution of (11) be resumed with the new initial conditions:

$$\left.\begin{aligned} \theta_{\tau+o} &= \theta_M : \text{ maximum value of } \theta \\ \dot{\theta}_{\tau+o} &= \left(1 - \frac{l_1^2(1+e)}{k_B^2 + l_g^2}\right) \dot{\theta}_{\tau-o} \end{aligned}\right\} \quad (12)$$

In the equations (12), the subscript τ denotes the instant at which the bourrelet strikes the land top, $\tau - o$ the instant just before striking and $\tau + o$ the instant just after striking. The time measurement, as before, has its origin at the initiation of the explosion and the time at which the band disengages is denoted by Υ. In consequence, the bourrelet describes a quasi-cycloidal motion along the land top which is terminated by the emergence of the bourrelet from the muzzle.



ANGLE - TIME RELATION INSIDE BORE OF THE GUN

Fig. 4

It is shown in the source cited that J, Υ, Ψ and Ψ may be calculated by:

$$J = \frac{l_g}{v_0}\sqrt{\dot\theta^2 + \theta^2\dot\psi^2} \tag{13}$$

$$\gamma = \arc\sin\left(\frac{-\dot\theta\cos\psi + \theta\sin\psi\,\dot\psi}{\sqrt{\dot\theta^2 + \theta^2\dot\psi^2}}\right) \tag{14}$$

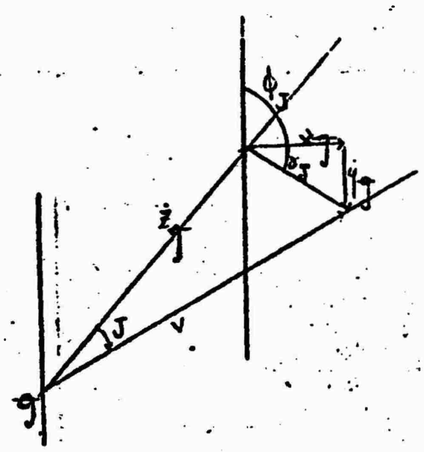
$$\psi = \psi_0 + \frac{2\pi}{nd_H} z_b \tag{15}$$

$$\dot\psi = \frac{2\pi}{nd_H}\dot z_b. \tag{16}$$

The quantity z_b is the travel of the band and $\dot z_b$ its time derivative. Capital letters denote the values of these quantities at the instant of disengagement of the band. The quantity $\dot Z_b$ may be approximated by v_g. Therefore:

$$\Psi = \psi_0 + \frac{2\pi}{nd_H} Z_b \tag{17}$$

$$\dot\Psi = N_\theta = \frac{2\pi}{nd_H} v_0. \tag{18}$$



JUMP ON EMERGENCE DUE TO YAW DUE TO CLEARANCE AND ITS ORIENTATION

Fig. 5

For many types of shell, the impulses of the blows received by the bourrelet on impact with the land top are very small.

The condition that these impulses are small is expressed by:

$$l_1 \longrightarrow \sqrt{\frac{k_B^2 + l_g^2}{1 + e}} \, . \tag{19}$$

The quantity l_l is the perpendicular distance in the shell from the point of impact on the bourrelet to the diametral plane through the band; the quantity l_g is the distance from the center of mass to the band plane; the quantity k_B is the radius of gyration of the shell about a transverse axis through the center of mass and the quantity e is the coefficient of restitution of the bourrelet. If the relation (19) is approximately satisfied and θ_o is as great as a few seconds of arc, the angle θ attains its maximum, θ_M, before nine-tenths of the time of travel. Thereafter the bourrelet remains nearly in contact with the land-tops: that is, all the maximum ordinates of the quasi-cycloid described by the bourrelet are very small. After the time of the first impact, τ, it may be considered that

$$\left.\begin{aligned} &\theta_M - \delta_\theta \leq \theta \leq \theta_M \\ &-\delta_{\dot\theta} \leq \dot\theta \leq \delta_{\dot\theta} \end{aligned}\right\} \tag{20}$$

where δ_θ is an angle which is negligible in comparison with θ_M and $\delta_{\dot\theta}$ is an angular velocity which is negligible in comparison with $\theta_M \dot\Psi$. The employment of (20) results in the approximate equalities:

$$\left.\begin{aligned} &J = \frac{l_g}{v_0} \theta_M \dot\Psi \\ &\gamma = \text{arc sin}\,(\sin \Psi) = \Psi \, . \end{aligned}\right\} \tag{21}$$

Substitution of (21) in (10) results in:

$$\left.\begin{aligned} \mathfrak{G}_c &= \theta_{\mathcal{M}} \sqrt{1 + \left(\frac{l_g \dot{\Psi}}{v_o}\right)^2} \\ \phi_{\mathfrak{G}_c} &= \Psi - \text{arc sin} \left(\frac{\frac{l_g \dot{\Psi}}{v_o}}{\sqrt{1 + \left(\frac{l_g \dot{\Psi}}{v_o}\right)^2}} \right) . \end{aligned}\right\} \quad (22)$$

Simultaneous reduction of (21) and (22) with (17) and (18) yields:

$$\left.\begin{aligned} \mathfrak{G}_c &= \theta_{\mathcal{M}} \sqrt{1 + \left(\frac{2\pi l_g}{nd_H}\right)^2} \\ \phi_{\mathfrak{G}_c} &= \psi_o + \frac{2\pi}{nd} Z_b - \text{arc sin} \left(\frac{\left(\frac{2\pi l_g}{nd_H}\right)}{\sqrt{1 + \left(\frac{2\pi l_g}{nd_H}\right)^2}} \right) \\ J &= \theta_{\mathcal{M}} \left(\frac{2\pi l_g}{nd_H}\right) \\ \gamma &= \psi_o + \frac{2\pi}{nd_H} Z_b . \end{aligned}\right\} \quad (23)$$

Now $\left(\frac{2\pi l_g}{nd_H}\right)$ is small in the sense that its square is negligible

in comparison with unity and that

$$\sin\left(\frac{2\pi l_g}{nd_H}\right) = \frac{2\pi l_g}{nd_H}\ .^{1}$$

Then:

$$\left.\begin{aligned}
\mathfrak{E}_c &= \theta_M \\
\varphi_{\mathfrak{E}_c} &= \psi_o + \frac{2\pi}{nd_H} Z_b - \frac{2\pi}{nd} l_g = \psi_o + \frac{2\pi}{nd} Z_g \\
J &= \theta_M \left(\frac{2\pi l_g}{nd_H}\right) \\
\Upsilon &= \psi_o + \frac{2\pi}{nd_H} Z_b\ .
\end{aligned}\right\} \quad (24)$$

The quantities J and Υ are constants with respect to the time. The quantities $\mathfrak{E}_c$ and $\varphi_{\mathfrak{E}_c}$ represent the yaw on emergence of the band due to clearance and the orientation of that yaw at the same instant, Υ. Time measurements in exterior ballistics are based on an origin at the instant the center of mass passes the muzzle. Any quantity used as an initial value in exterior ballistics must correspond to the instant the center of mass emerges rather than the instant of disengagement of the band. The values $\mathfrak{E}_c$ and $\varphi_{\mathfrak{E}_c}$ must accordingly be modified to those values which can be used as initial conditions for the differential equations governing the motion of the axis of the shell exterior to the gun. The time interval during which the modification takes place is very small and it will be permissible to regard the velocity of the center of mass during this interval as constant and equal to v_o, the muzzle velocity. In consequence an extrapolation of $\mathfrak{E}_c$ and $\varphi_{\mathfrak{E}_c}$ backward in time

[1] In the case of the shell examined in the present report $\left(\frac{2\pi l_g}{nd_H}\right) = 0.25$.

from γ to $\gamma - \frac{\ell_g}{v_o}$ will be performed, since the interval from the emergence of the center of mass to the emergence of the band is sensibly equal to $\frac{\ell_g}{v_o}$. The quantities denoting the yaw and its orientation on emergence will be denoted by ε_c and φ_{ε_c}. The computed quantities may be illustrated by the following scheme.

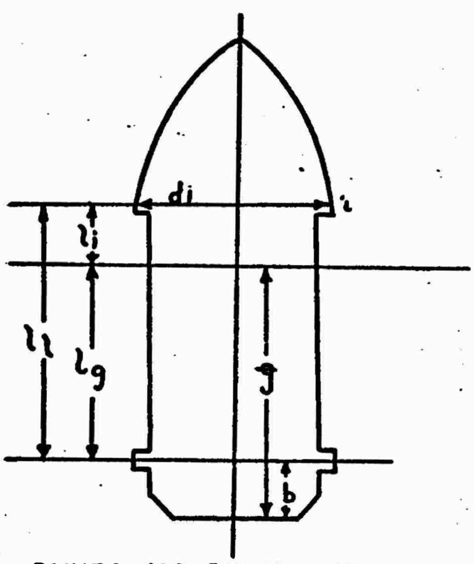
	Explosion Starts	Center of Mass Emerges	Band Emerges
Time with origin at the instant of initiation of the explosion	0	$(\gamma - \frac{\ell_g}{v_o})$	γ
Time with origin coincident with the emergence of the center of mass	-	0	$+ \frac{\ell_g}{v_o}$
θ	θ_o	$\longrightarrow$	$\doteq \theta_{\mathcal{M}}$
ψ	ψ_o	$\longrightarrow$	ψ
$\dot{\theta}$	0	$\longrightarrow$	$\doteq 0$
$\dot{\psi}$	0	$\longrightarrow$	$\dot{\psi}$
J		J	
$\varphi_J = \gamma + \frac{\pi}{2}$		φ_J	
Yaw on Emergence		ε_c	σ_c
Orientation of the Yaw on Emergence		φ_{ε_c}	φ_{σ_c}

It follows from the second of inequalities (20) and the consideration that $\delta_{\dot{\theta}}$ is an angular velocity which is negligible in comparison with $\theta_{\mathcal{M}\psi}$ that there is no change in the magnitude of the yaw during the time interval $\frac{\ell_g}{v_o}$, and therefore $\varepsilon_c = \sigma_c = \theta_{\mathcal{M}}$. However the orientation of this yaw changes because of its time derivative. Under the assumptions that have been made the time derivative may be equated to $\frac{2\pi v_o}{nd_H}$. The change in the orientation must be subtracted from δ_{σ_c} to obtain δ_{ε_c}, since the center of mass emerges before the band. This change

is $(\frac{2\pi v_o}{nd_H})\ \frac{l_g}{v_o}$. It has been noted $\varphi_J = \gamma + \frac{\pi}{2}$. Further Z_g, the total travel of the center of mass and Z_b, the total travel of the band are related by $Z_g = Z_b - l_g$. Therefore:

$$\left.\begin{aligned}
\varepsilon_c &= \theta_{\mathcal{M}} \\
\varphi_{\varepsilon_c} &= \psi_o + \frac{2\pi}{nd_H}(Z_g - l_g) \\
J &= \theta_{\mathcal{M}}\left(\frac{2\pi l_g}{nd_H}\right) \\
\varphi_J &= \frac{\pi}{2} + \psi_o + \frac{2\pi}{nd_H}(Z_g + l_g).
\end{aligned}\right\} \quad (25)$$

An equation, which will now be derived, will express $\theta_{\mathcal{M}}$, the maximum value of θ, in terms of the dimensions of the shell and the diameter of the bore of the gun. The derivation appears worthwhile because of the important consequences that attach to the equation.



POINTS AND DIMENSIONS OF SHELL

Fig. 6

Let d_i, the diameter of the bourrelet at the point of impact be taken as nearly equal to d, the master diameter of the shell. Let the shell be supposed centered at the band by the band[1] By the definitions suggested by the dimensions of the shell, $l_g = g - b$ and $l_l = l_g + l_i$. The quantity g is the distance of the center of mass from the base, b is the band distance from the base and l_i is the distance from the center of mass to the diametral plane through the point on the shell where the impact of the bourrelet takes place. Let d_H denote the diameter of the bore between the land tops.

[1] It was suggested by Dr. L. S. Dederick that the ramming of the shell induces approximate centering of the shell at the band.

It will be presumed that the impacts of the bourrelet always take place on a land top. By geometry:

$$\sin \theta_M = \frac{\frac{d_H}{2} - \frac{d_i}{2}\cos\theta_M}{l_i} . \qquad (26)$$

Since θ_M is extremely small, let $\sin \theta_M$ be replaced by θ_M and $\cos \theta_M$ be replaced by unity in equation (26). Then:

$$\theta_M = \frac{d_H - d_i}{2l_i} = \frac{d_H - d}{2l_i} . \qquad (27)$$

Introduction of the definitions of l_g and l_i into equations (25) leads to:

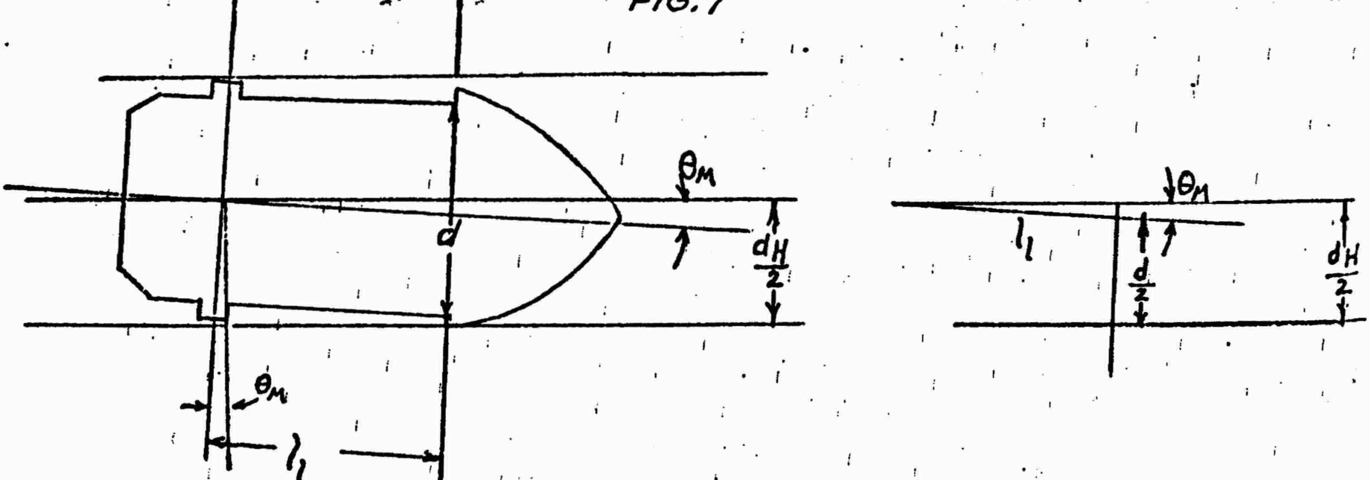
$$\varepsilon_c = \frac{(d_H - d)}{2l_i} = \frac{d_H - d}{2(l_i + g - b)}$$

$$\varphi_{\varepsilon_c} = \psi_o + \frac{2\pi}{nd_H}(Z_g - g + b) = \psi_o + \frac{2\pi}{nd_H}(Z_g - l_g)$$

$$J = \frac{(d_H - d)}{2(l_i + g - b)}\left(\frac{2\pi\,[g - b]}{nd_H}\right) = \frac{\pi}{n}\left(\frac{d_H - d}{d_H}\right)\left(\frac{g - b}{l_i + g - b}\right) = \frac{\pi}{n}\left(\frac{d_H - d}{d_H}\right)\frac{l}{l_i}$$

$$\varphi_J = \frac{\pi}{2} + \psi_o + \frac{2\pi}{nd_H}(Z_g + g - b) = \frac{\pi}{2} + \psi_o + \frac{2\pi}{nd_H}(Z_g + l_g).$$

FIG. 7



MAXIMUM ANGLE BETWEEN THE AXIS OF THE SHELL AND THE AXIS OF THE BORE INSIDE THE GUN.

Let it be supposed that the normal value of ε_c is ε_{c_o}, the normal value of φ_{ε_c} is $\varphi_{\varepsilon_{c_o}}$, the normal value of J is J_o and the normal value of Φ_J is Φ_{J_o}. The angle of departure, Φ, the side jump, i_1, the initial yaw, δ_o, and its orientation, φ_{δ_o}, are the initial conditions which are modified by the action of ε_{L_i}: the general values of these quantities are given by:

$$\left.\begin{aligned} \Phi &= E + j + J\cos\varphi_J \\ I &= i_1 + J\sin\varphi_J \\ \delta_o &= \varepsilon_c \\ \varphi_{\delta_o} &= \varphi_{\varepsilon_c}. \end{aligned}\right\} \quad (29)$$

Differentiation of equations (29) results in:

$$\left.\begin{aligned} \frac{\partial\Phi}{\partial L} &= \frac{\partial J}{\partial L}\cos\varphi_J - J\sin\varphi_J\frac{\partial\varphi_J}{\partial L} \\ \frac{\partial I}{\partial L} &= \frac{\partial J}{\partial L}\sin\varphi_J + J\cos\varphi_J\frac{\partial\varphi_J}{\partial L} \\ \frac{\partial\delta_o}{\partial L} &= \frac{\partial\varepsilon_c}{\partial L} \\ \frac{\partial\varphi_{\delta_o}}{\partial L} &= \frac{\partial\varphi_{\varepsilon_c}}{\partial L} \end{aligned}\right\} \quad (30)$$

Hence:

$$
\left.
\begin{aligned}
\Delta_L\Phi &= \left(\frac{\partial\Phi}{\partial L}\right)_o \varepsilon_L = \cos\varphi_{J_o}\left(\frac{\partial J}{\partial L}\right)_o \varepsilon_L - J_o \sin\varphi_{J_o}\left(\frac{\partial\varphi_J}{\partial L}\right)_o \varepsilon_L \\
\Delta_L I &= \left(\frac{\partial I}{\partial L}\right)_o \varepsilon_L = \sin\varphi_{J_o}\left(\frac{\partial J}{\partial L}\right)_o \varepsilon_L + J_o \cos\varphi_{J_o}\left(\frac{\partial\varphi_J}{\partial L}\right)_o \varepsilon_L \\
\Delta_L\delta_o &= \left(\frac{\partial\mathcal{E}_c}{\partial L}\right)_o \varepsilon_L \\
\Delta_L\Phi\delta_o &= \left(\frac{\partial\varphi_{\mathcal{E}_c}}{\partial L}\right)_o \varepsilon_L \, .
\end{aligned}
\right\}(31)
$$

The partial derivatives occurring in equations (31) are supplied for any particular ε_L from the equations (28). The range, deflection and dispersion effects due to the action of ε_L during the different regimes will now be obtained. The quantities sought are the values of the total derivatives $\frac{dX}{dL}$ and $\frac{dZ}{dL}$. When these quantities have been found, the right hand sides of the equations (6) and (9) can be evaluated. The formulae for the effects will have the form:

$$
\left.
\begin{aligned}
\Delta_L X &= \left(\frac{dX}{dL}\right)\varepsilon_L = c_L\varepsilon_L \\
\Delta_L Z &= \left(\frac{dZ}{dL}\right)\varepsilon_L = k_L\varepsilon_L \\
\Delta_L \sigma_X^2 &= \left(\frac{dX}{dL}\right)^2 \Delta\sigma_L^2 = c_L^2\,\Delta\sigma_L^2 \\
\Delta_L \sigma_Z^2 &= \left(\frac{dZ}{dL}\right)^2 \Delta\sigma_L^2 = k_L^2\,\Delta\sigma_L^2 \, .
\end{aligned}
\right\}(32)
$$

Since the latter two equations of (32) will not be employed in their present form, attention will be confined temporarily to the first two.

The action of ε_L may be considered as divided into two parts: the modification of initial conditions at the muzzle of the gun and the modification of the form of the equations of motion beyond the muzzle of the gun. In order to develop this principle let the action of ε_L be traced through the various regimes. The initial conditions for the second regime have been modified by the action of ε_L during the first regime. Let the effects of changes in initial conditions and the effects of a change in the form of the equations of motion be considered small and independent. Let the effects due to changes in the form of the equations of motion due to the action of ε_L be written as $\left(\frac{\partial X}{\partial p}\frac{\partial p}{\partial \varepsilon_L}\right)\varepsilon_L$ and $\left(\frac{\partial Z}{\partial q}\frac{\partial q}{\partial \varepsilon_L}\right)\varepsilon_L$. These quantities will be interpreted later and for the present the expressions within the parentheses may be considered as single quantities such that:

$$\left.\begin{aligned} \left(\frac{\partial X}{\partial p}\cdot\frac{\partial p}{\partial \varepsilon_L}\right) &= \lim_{\varepsilon_L\to 0}\left(\frac{X_p - X}{\varepsilon_L}\right) \\ \left(\frac{\partial Z}{\partial q}\cdot\frac{\partial q}{\partial \varepsilon_L}\right) &= \lim_{\varepsilon_L\to 0}\left(\frac{Z_q - Z}{\varepsilon_L}\right). \end{aligned}\right\} \quad (33)$$

In the equations (33), X and Z denote the values of the elements for normal shell and X_p and Z_q the elements obtained with the same initial conditions but with equations of motion differing from the normal equations by reason of the action of ε_L. The effects upon the elements X and Z of the changed initial conditions for the normal equations of motion must be added to the effects of the changed form of the equations of motion in order to determine the total effects, $\Delta_L X$ and $\Delta_L Z$. The effects of ε_L upon the initial conditions are given by (31). Then the total effects upon the elements X and Z are given by:

$$\left.\begin{aligned}
\Delta_L X &= \left(\frac{dX}{dL}\right)\varepsilon_L = \left[\frac{\partial X}{\partial \phi}\frac{\partial \phi}{\partial L} + \frac{\partial X}{\partial I}\frac{\partial I}{\partial L} + \frac{\partial X}{\partial \mathcal{E}_c^2}\frac{\partial \mathcal{E}_c^2}{\partial L} + \frac{\partial X}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial L}\right. \\
&\qquad \left. + \frac{\partial X}{\partial p}\frac{\partial p}{\partial \varepsilon_L}\right]\varepsilon_L \\
\Delta_L Z &= \left(\frac{dZ}{dL}\right)\varepsilon_L = \left[\frac{\partial Z}{\partial \phi}\frac{\partial \phi}{\partial L} + \frac{\partial Z}{\partial I}\frac{\partial I}{\partial L} + \frac{\partial Z}{\partial \mathcal{E}_c^2}\frac{\partial \mathcal{E}_c^2}{\partial L} + \frac{\partial Z}{\partial \varphi_0}\frac{\partial \varphi_0}{\partial L}\right. \\
&\qquad \left. + \frac{\partial Z}{\partial q}\frac{\partial q}{\partial \varepsilon_L}\right]\varepsilon_L \, .
\end{aligned}\right\} (34)$$

The first four terms of each of the equations (34) represent the change in the element due to the action of ε_L in changing the initial conditions for the normal equations of motion for the second and thereby later regimes. The last terms of each of these equations represent the change in the element due to the action of ε_L in changing the form of the equations of motion. The component terms in the effects due to changed initial conditions are now clearly recognizable: they comprise the effects due to the action of ε_L inside the bore of the gun. The separate terms are due to the action of ε_L in the bore in modifying the final angle of departure, side-jump in the slant plane, yaw on emergence due to yaw due to clearance and the orientation of this yaw. The separate partial derivatives are not all easily computed; the expressions must be so modified as to yield only terms which can be determined on theoretical grounds. It will be observed that:

$$\left.\begin{aligned}
\frac{\partial X}{\partial I} &\doteq 0 \\
\frac{\partial Z}{\partial \phi} &\doteq 0 \\
\frac{\partial X}{\partial \phi} &= \frac{\partial X}{\partial E} \\
\frac{\partial Z}{\partial I} &= X \sec E \, .
\end{aligned}\right\} (35)$$

The last two quantities in equations (35) are readily obtainable from firing tables. The action of ε_L over the trajectory exterior to the muzzle consists in a change in the form of the equations of motion. The change in the form of the equations of motion results, in part, from the presence of additional terms not present in the equations of motion for normal shell, and, in part, from the modification of the aerodynamic coefficients occurring in the normal equations of motion. The action of ε_L in the bore results, in part, in changing the orientation of the yaw on emergence which results in a change in the direction of the cross-wind force on the shell. This change in the direction of the cross-wind force results in a change in the displacement of the trajectory due to yaw. This change in the displacement of the trajectory affects the elements X and Z somewhat, but this effect will be considered negligible in the present instance. Then the action of ε_L in changing the orientation of the yaw on emergence will be presumed small in so far as it affects the range and deflection through the change in the direction of the cross-wind force.

On examination of (28), it appears that φ_{ε_c} will be changed if ψ_o, n, d_H, Z_g, g or b are changed. Then if ε_L denotes ε_g or ε_b an effect on φ_{ε_c} will appear due to the action of ε_L. The effect on φ_o in these two cases is:

$$\left.\begin{aligned} \Delta_g\varphi_{\varepsilon_c} &= -\frac{2\pi}{nd_H}\varepsilon_g \\ \Delta_b\varphi_{\varepsilon_c} &= \frac{2\pi}{nd_H}\varepsilon_b \end{aligned}\right\} \quad (36)$$

It is clear that ε_g or ε_b will ordinarily not exceed one half inch. In this case $\Delta_{\varepsilon_L}\ \varphi_{\varepsilon_c}$ will not exceed 0.025 radian or 1°5 in the case of 155mm.Howitzer H.E. Shell Mk.I. It is improbable that a change in the orientation of the yaw on emergence of 1°5 is capable of changing the direction of the cross-wind force due to yaw due to conditions of launching sufficiently to modify the displacement of the trajectory appreciably during the second and third regimes.

The fractional effects of $\phi_{\mathcal{E}_c}$ in the fourth regime are all zero. then:

$$\left.\begin{aligned} \frac{\partial X}{\partial \phi_\theta} \frac{\partial \phi_{\mathcal{E}_c}}{\partial L} \quad \varepsilon_L &\doteq 0 \\ \frac{\partial Z}{\partial \phi_\theta} \frac{\partial \phi_{\mathcal{E}_c}}{\partial L} \quad \varepsilon_L &\doteq 0 . \end{aligned}\right\} \quad (37)$$

By means of equations (35) and (37) equations (34) are reducible to the form:

$$\left.\begin{aligned} \Delta_L X &= \left[\frac{\partial X}{\partial E} \frac{\partial \phi}{\partial L} + \frac{\partial X}{\partial \mathcal{E}_c^2} \frac{\partial \mathcal{E}_c^2}{\partial L} + \frac{\partial X}{\partial p} \frac{\partial p}{\partial \varepsilon_L} \right] \varepsilon_L \\ \Delta_L Z &= \left[X \sec E \frac{\partial I}{\partial L} + \frac{\partial Z}{\partial \mathcal{E}_c^2} \frac{\partial \mathcal{E}_c^2}{\partial L} + \frac{\partial Z}{\partial q} \frac{\partial q}{\partial \varepsilon_L} \right] \varepsilon_L . \end{aligned}\right\} \quad (38)$$

Let the second regime of the motion be considered: the trajectory between the emergence of the band and the first maximum yaw can be approximated by a straight line traversed with uniform speed. Then the action of ε_L during the second regime consists entirely in modifying the first maximum yaw due to the effect of ε_L on $\mathcal{E}_c$. The yaw, $\mathcal{E}_c$, is an initial condition for the second regime. It is worthy of note that the maximum amplitude of yaw near the gun, α_0, may be calculated from the yaw on emergence due to clearance, $\mathcal{E}_c$, by an important formula due to R. H. Kent and H. P. Hitchcock:[1]

$$\alpha_0 = \left(\frac{2B}{A} - 1\right)\left(\frac{S_0}{S_0 - 1}\right)^{1/2} \mathcal{E}_c . \quad (39)$$

It would be possible to use this formula for some of the following deductions if a relation were given between α, the amplitude of yaw at any time, and α_0, the amplitude of yaw near the gun. Another procedure is preferable. Substitution of the first of equations (30) into equation (38) leads to:

[1] Effect of Cross Wind on Yaw.

$$\left.\begin{aligned}\Delta_L X &= \left[\cos\varphi_J \frac{\partial X}{\partial E}\frac{\partial J}{\partial L} - \sin\varphi_J \frac{\partial X}{\partial E}\frac{\partial \varphi_J}{\partial L} + \frac{\partial X}{\partial \mathcal{E}_c^2}\frac{\partial \mathcal{E}_c^2}{\partial L} + \frac{\partial X}{\partial p}\frac{\partial p}{\partial \varepsilon_L}\right]\varepsilon_L \\ \Delta_L Z &= \left[\sin\varphi_J \sec EX \frac{\partial J}{\partial L} + \cos\varphi_J \sec EX \frac{\partial \varphi_J}{\partial L} + \frac{\partial Z}{\partial \mathcal{E}_c^2}\frac{\partial \mathcal{E}_c^2}{\partial L} + \frac{\partial Z}{\partial q}\frac{\partial q}{\partial \varepsilon_L}\right]\varepsilon_L .\end{aligned}\right\} \quad (40)$$

The motion between the time of the first maximum yaw and the time when the yaw has become dominantly precessional is comprised in the third regime. In this regime the curvature of the trajectory is small, and the yaw is dominantly due to conditions of launching. There is some small change in the normal displacement of the trajectory due to the change in the cross wind force arising from the change in the magnitude of the maximum yaw. This change in the magnitude of the normal displacement of the trajectory results from the action of ε_L during the first two regimes in changing the maximum yaw; it may be considered negligible. The initial amplitude of yaw, α_o, is the only initial condition for the third regime which is modified by the previous action of ε_L. To some approximation during this regime, then, the effect of ε_L consists in modifying the yaw-time relation:[1]

$$\delta = \alpha \sin \left(\frac{AN}{B}\right)\left(\frac{S-1}{S}\right)^{1/2} t \; . \qquad (41)$$

The quantities given in equation (41) are computed from

$$\alpha = J_1 \frac{\left(\frac{S_o - 1}{S_o}\right)^{1/4}}{\left(\frac{S-1}{S}\right)^{1/4}} e^{-h_1} \cosh(j_1 - h_2) \qquad (42)$$

$$j_1 = \tanh^{-1}\left[\frac{\left(\frac{S_o - 1}{S_o}\right)^{1/2}}{\left(\frac{2B}{A} - 1\right)}\right] . \qquad (43)$$

[1] This equation and the definitions given below are those developed by: R.H.Kent: "An Elementary Treatment of the Motion of a Spinning Projectile About its Center of Gravity," Ballistic Research Laboratory Report No. 85.

$$J_1 = \frac{\mathcal{E}_c}{\sinh j_1} \cdot \tag{44}$$

$$h_1 = \frac{1}{2}\int_0^t (f + k)\, dt \cdot \tag{45}$$

$$h_2 = \frac{1}{2}\int_0^t (f - k)\left(\frac{S}{S-1}\right)^{1/2} dt \cdot \tag{46}$$

$$f = \frac{\rho d^4 v K_H}{B} \cdot \tag{47}$$

$$k = \frac{\rho d^2 v K_L}{m} \cdot \tag{48}$$

$$S = S_0 \left(\frac{v_0}{v}\right)^2 \cdot \tag{49}$$

Over a number of complete periods, the average value of the ratio $\left(\frac{\delta^2}{\alpha^2}\right)$ with respect to time is one-half. Then:

$$\left.\begin{aligned} \overline{\delta^2} &= \frac{J_1^2}{2}\left(\frac{S_0 - 1}{S_0}\right)^{1/2}\left(\frac{S}{S-1}\right)^{1/2} e^{-2h_1} \cosh^2(j_1 - h_2) \\ &= \frac{\mathcal{E}_c^2}{2\sinh^2 j_1}\left(\frac{S_0 - 1}{S_0}\right)^{1/2}\left(\frac{S_0 v_0^2}{S_0 v_0^2 - v^2}\right)^{1/2} e^{-2h_1} \cosh^2(j_1 - h_2). \end{aligned}\right\} \tag{50}$$

In the terminology of R. H. Kent, the yaw in the third regime of the motion is described as the yaw due to conditions of launching. By equation (50) it is shown that the average square

of this yaw is proportional to the product of the square of $\mathcal{E}_c$ and a function of the time. Let the constant j_1 be supposed small:[1] then, approximately:

$$j_1 \doteq \sinh j_1 \doteq \tan h j_1 \, . \qquad (51)$$

The equation (50) becomes:

$$\delta^2 = \frac{\mathcal{E}_c^2}{2}\left(\frac{2B}{A} - 1\right)^2\left(\frac{S_o}{S_o - 1}\right)^{1/2}\left(\frac{S_o v_o^2}{S_o v_o^2 - v^2}\right)^{1/2} e^{-2h_1} \cos h^2(j_1 - h_2)$$

$$= \frac{\alpha_o^2}{2}\left(\frac{S_o - 1}{S_o}\right)^{1/2}\left(\frac{S_o v_o^2}{S_o v_o^2 - v^2}\right)^{1/2} e^{-2h_1} \cos h^2(j_1 - h_2)$$

$$= \frac{\alpha_o^2}{2}\left(\frac{S_o v_o^2 - v_o^2}{S_o v_o^2 - v^2}\right)^{1/2} e^{-2h_1} \cos h^2(j_1 - h_2) \, . \qquad (52)$$

In general the total drag of a yawing projectile, $D(\delta)$, is related to the drag for zero yaw, $D(0)$, by the formula:[2]

$$D(\delta) = D(0)(1 + K_{D_\delta}\delta^2) \, . \qquad (53)$$

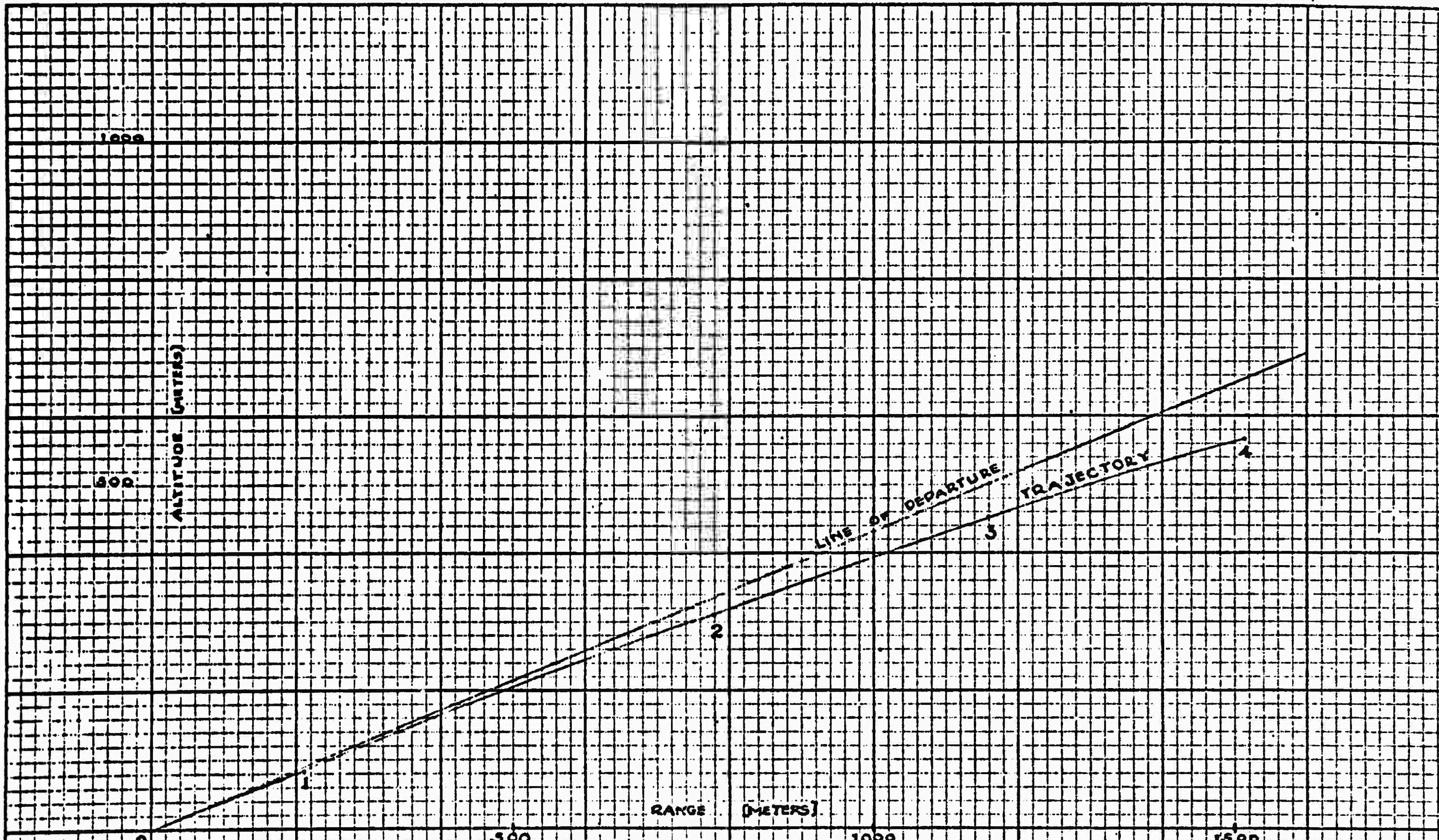
The quantity K_{D_δ} is described as the yaw drag coefficient: evidently the augmentation of drag due to yaw is $K_{D_\delta}\delta^2(D(0))$.

The curvature of the trajectory during the third regime is small: the early portion of the normal trajectory for the shell examined in the present report is shown in Fig. 8. To a degree of approximation sufficient for the computation of some differential effects during this regime:

$$\theta_1 \doteq \phi = \text{arc tan}\,\frac{\dot{y}_o}{\dot{x}_o} = \text{arc sin}\,\frac{\dot{y}_o}{v_o} \, . \qquad (54)$$

[1] In the case of 155mm. Howitzer H.E. Shell Mk.I fired at $v_o' = 1478$ [FT/SEC], the values are : $j_1 = 0.04059$, $\sin hj_1 = 0.04059$, $\tan hj_1 = 0.04057$.

[2] H.P.Hitchcock: "Aerodynamic Formulae and Nomenclature," Ballistic Research Laboratory Report 111. Hitchcock has found that an average of K_{D_δ} for different shell is 0.005 where δ is given in degrees.



INITIAL PORTION OF NORMAL TRAJECTORY OF 155mm HOWITZER,
H.E. SHELL MK.I FIRED AT 1478 [FT/SEC] AT AN
ELEVATION OF 23° 15'

Fig 8

For the normal trajectory under standard ballistic table conditions, the velocity of the center of mass of the shell satisfies the differential equation:

$$\frac{dv_N}{dt} = -\frac{D(0)}{m} - g \sin\theta_1 = -\frac{GHv}{C} - g \sin\theta_1 . \qquad (55)$$

For yawing shell, if the cross wind force be neglected:

$$\frac{dv_\delta}{dt} = -\frac{D(\delta)}{m} - g \sin\theta_1 = -\frac{D(0)}{m}(1 + K_{D_\delta}\delta^2) - g \sin\theta_1$$

$$= -\frac{GHv}{C}(1 + K_{D_\delta}\delta^2) - g \sin\theta_1 . \qquad (56)$$

The effect on velocity due to yaw, $\Delta_\delta v$, at any point on the trajectory is given by:

$$\Delta_\delta v = v_\delta - v_N . \qquad (57)$$

Let the second order effects of δ on v through δ on θ_1, of δ on v through δ on G through δ on v, and of δ on H through δ on y through δ on $\dot{y}$ be neglected. The effect of yaw on velocity will then satisfy the differential equation:

$$\frac{d\Delta_\delta v}{dt} = -\frac{GHv}{C} K_{D_\delta}\delta^2 = -K_{D_\delta}\delta^2 E_N v . \qquad (58)$$

The integration of (58) yields the drop in velocity up to any time, t, due to yaw due to conditions of launching.

$$\Delta_\delta v = -K_{D_\delta} \int_0^t \delta^2 E_N v\,dt \qquad (59)$$

The third regime of the motion is traversed rapidly since the velocity is great in this portion of the trajectory. The yaw due to conditions of launching is completely damped out within a small fraction of the time of flight; the third regime lasts only about one tenth the total time of flight in the case of the present shell. In consequence, the drop in velocity due to yaw due to conditions of launching may be treated, without considerable error, as an effect on muzzle velocity. The effect on range of this drop in initial velocity is:

$$\Delta_{v_\delta} X = -\left[K_{D_\delta}\int_0^t Ev\delta^2 dt\right]\left(\frac{\partial X}{\partial v_o}\right). \tag{60}$$

Substitution of (52) into (60) results in:

$$\Delta_{v_\delta} X = -\left[\frac{K_{D_\delta}}{2}\left(\frac{2B}{A}-1\right)^2\left(\frac{S_o}{S_o-1}\right)^{1/2}\int_0^t Ev\left(\frac{S_o v_o^2}{S_o v_o^2 - v_2}\right)^{1/2} e^{-2h_1}\cosh^2(j_1-h_2)dt\right]$$

$$\mathcal{E}_c^2\left(\frac{\partial X}{\partial v_o}\right). \tag{61}$$

The equation (61) gives the total range effect due to the drop in velocity due to an abnormal initial yaw $\mathcal{E}_c$. Only the effect of ε_L upon $\mathcal{E}_c$ and thereby upon α_o is desired at present. But $\Delta_{v_\delta} X$ is proportional to $\mathcal{E}_c^2$ and therefore:

$$\frac{\partial X}{\partial \mathcal{E}_c^2} = \frac{\Delta_{v_\delta} X}{\mathcal{E}_c^2}. \tag{62}$$

A certain type of mean of Ev is given by the equation:

$$\overline{Ev} = \frac{\int_0^t Ev\delta^2 dt}{\int_0^t \delta^2 dt}. \tag{63}$$

For the short period comprised in the third regime, it is likely that the mean (63) will agree to perhaps two significant figures with the arithmetic mean of Ev with respect to time over the third regime. Accordingly:

$$\Delta_{v_{\Delta_{\varepsilon_c}}} X = \Bigg[- \frac{K_{D_\delta}}{2}\left(\frac{2B}{A} - 1\right)^2 \left(\frac{S_o}{S_o - 1}\right)^{1/2} Ev$$

$$\int_o^t \left(\frac{S_o v_o^2}{S_o v^2{}_o - v^2}\right)^{1/2} e^{-2h_1} \cosh^2(j_1 - h_2)dt \Bigg] \left(\frac{\partial X}{\partial v_o}\right) \left(\frac{\partial \varepsilon_c^2}{\partial L}\right) \varepsilon_L. \tag{64}$$

Evidently (64) is completely equivalent to:

$$\Delta_{\mathcal{E}_c} X = \frac{\partial X}{\partial \mathcal{E}_c^2} \frac{\partial \mathcal{E}_c^2}{\partial L} \varepsilon_L$$

of which the right hand side is the third term of the first equation of (40). The third term of the second equation of (40) is:

$$\frac{\partial Z}{\partial \mathcal{E}_c^2} \frac{\partial \mathcal{E}_c^2}{\partial L} \varepsilon_L$$

which is the effect on deflection of the action of ε_L in changing $\mathcal{E}_c$. This effect has attained its complete magnitude by the end of the third regime in consequence of the fact that the yaw due to conditions of launching is damped out by the end of the third regime. The dominant parts of the normal deflection are the drift and the deflection due to the side-jump. The drift arises from the action of the cross-wind force due to the precessional yaw during the fourth regime. The side-jump is an initial condition for the second regime. The effect on the deflection due to the action of ε_L in modifying the side jump I has been accounted for. It has a value given by the term $X \sec E \frac{\partial I}{\partial L} \varepsilon_L$ of equation (38) or its equivalent, the first two terms of the second of equations (33). Therefore the term

$$\frac{\partial Z}{\partial \mathcal{E}_c^2} \frac{\partial \mathcal{E}_c^2}{\partial L} \varepsilon_L$$

arises from the action of ε_L in modifying the deflection component of the displacement of the trajectory during the third regime. This modification of the displacement is due to the augmentation of the cross-wind force arising from the augmentation of δ by the increase in its amplitude. If the trajectory were nearly straight, this effect along the trajectory would nearly cancel, the effect during each half period annihilating the effect arising in the half period preceding. This

is the condition in the third regime since the curvature of the trajectory and the consequent precessional yaw and drift arising therefrom are extremely small. Therefore both this whole displacement and the effect on it of the augmentation of δ are negligible. Dropping this deflection term and substituting (64) for the range term in equations (40) results in:

$$\left.\begin{aligned}
\Delta_L X = \Bigg[& \cos\varphi_J J \frac{\partial X}{\partial E}\frac{\partial J}{\partial L} - \sin\varphi_J J \frac{\partial X}{\partial E}\frac{\partial \varphi_J}{\partial L} - \frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A} - 1\right)^2 \\
& \left\{\int_0^t \left(\frac{S_0^2 v_0^2}{(S_0 - 1)(S_0 v_0^2 - \bar{v}^2)}\right)^{1/2} e^{-2h_1}\cosh^2(j_1 - h_2)\,dt\right\} \\
& \frac{\partial X}{\partial v_0}\frac{\partial \varepsilon_c^2}{\partial L} + \frac{\partial X}{\partial p}\frac{\partial p}{\partial \varepsilon_L}\Bigg]\varepsilon_L \\
\Delta_L Z = \Bigg[& \sin\varphi_J \sec E X \frac{\partial J}{\partial L} + \cos\varphi_J \sec E X J \frac{\partial \varphi_J}{\partial L} + \frac{\partial Z}{\partial q}\frac{\partial q}{\partial \varepsilon_L}\Bigg]\varepsilon_L .
\end{aligned}\right\} \quad (65)$$

It is noteworthy that all terms so far obtained are strictly calculable on theoretical grounds if only derivatives of J, φ_J and $\mathcal{E}_c$ with respect to the abnormality in question can be found. The latter derivatives can be found explicitly if the abnormalities are with respect to the dimensions given in (28). Explicit derivatives can be found theoretically in a few other cases and experimentally in still other cases. These yield the first three terms of the first equation and the first two terms of the second equation of (65). These five terms are all initiated by the modification of the motion of the axis of the shell inside the bore of the gun due to the action of ε_L. These terms are calculable as if they were effects upon elements X and Z due to small changes of the initial conditions at the muzzle of the gun arising from the action of ε_L inside the bore of the gun. These five terms are the total effects of the changed initial conditions and they have attained their complete magnitude by the time of entrance of the projectile into the fourth regime.

The last term in each of equations (65) represents a quantity arising from the action of ε_L entirely outside the gun: these terms are due to the action of ε_L in modifying the drag and the cross-wind force upon the shell. In this sense the last term in each equation is not due to changed initial conditions at the muzzle of the gun but rather to the action of ε_L in changing the forces acting on the shell exterior to the gun whose action appears dominantly in the fourth regime. In contradistinction to the terms due to changed initial conditions the latter two terms in each equation arise from the presence of additional terms or a change in the form of the equations of motion exterior to the gun. Practically the whole of the trajectory is described in regimes three and four, and these consequences of ε_L are a change in the drag for zero yaw and a change in the permanent yaw. The change in the permanent yaw results in an augmentation of drag due to yaw and an augmentation of the cross-wind force. These may be considered as resulting in an effect upon the average ballistic coefficient and an effect upon the drift. Then:

$$\left.\begin{aligned} \frac{\partial X}{\partial p}\frac{\partial p}{\partial \varepsilon_L} &= \frac{C\partial X}{C\partial C}\frac{\partial C}{C\partial L} = \frac{\partial X}{\partial C}\frac{\partial C}{\partial L} \\ \frac{\partial Z}{\partial q}\frac{\partial q}{\partial \varepsilon_L} &= \frac{\partial \zeta}{\partial L}. \end{aligned}\right\} \quad (66)$$

Substitution of (66) into (65) leads to:

$$\left.\begin{aligned} \Delta_L X = \Bigg[& \left(\cos\varphi \frac{\partial J}{J\partial L} - \sin\varphi_J J \frac{\partial \varphi_J}{\partial L}\right)\frac{\partial X}{\partial E} \\ & - \frac{K_{D\delta}\overline{Ev}}{2}\left(\frac{2B}{A} - 1\right)^2 \left\{\int_0^t \left(\frac{S_o^2 v_o^2}{(S_o-1)(S_o v_o^2 - v^2)}\right)^{1/2} e^{-2h_1}\cos h^2(j_1-h_2)\,dt\right\} \\ & \frac{\partial X}{\partial v_o}\frac{\partial \varepsilon_c^2}{\partial L} + \frac{C\partial X}{\partial C}\frac{\partial C}{C\partial L}\Bigg]\varepsilon_L \\ = \Delta_{J_L} X & + \Delta_{\varphi_{J_L}} X + \Delta_{v_{o_L}} X + \Delta_{C_L} X. \end{aligned}\right\} \quad (6$$

$$\left.\begin{aligned} \Delta_L Z &= \left[\left(\sin \varphi_J \frac{\partial J}{\partial L} + \cos \varphi_J J \frac{\partial \varphi_J}{\partial L}\right) X \sec E + \zeta\left(\frac{\partial \zeta}{\zeta \partial L}\right) \right] \varepsilon_L \\ &= \Delta_{J_L} Z + \Delta_{\varphi_{J_L}} Z + \Delta_{\zeta_L} Z. \end{aligned}\right\} \quad (67)$$

The quantity $\frac{C \partial X}{\partial C}$ may be obtained by simple calculation from the density effect column in firing tables. The products $\frac{\partial C}{C \partial L} \varepsilon_L$ and $\frac{\partial \zeta}{\zeta \partial L} \varepsilon_L$ are obtainable with somewhat greater difficulty. At present for the general case these quantities must be obtained from experiment. In the case of an abnormality in mass or diameter the derivative $\frac{\partial C}{C \partial L}$ can be obtained explicitly from the formula:

$$C = \frac{m}{id^2}. \quad (68)$$

H. P. Hitchcock[1] has formulae expressing the results of experiment in the dependence of form factors upon various dimensions of the projectile. In case L is a quantity upon which Hitchcock has found the dependence in i, differentiation with respect to L of Hitchcock's formula for i will yield $\frac{\partial i}{\partial L}$. Then the formula:

$$\frac{\partial C}{C \partial L} = \frac{\partial C}{C \partial i} \frac{\partial i}{\partial L} \quad (69)$$

will provide this product. But:

$$\frac{\partial C}{C \partial i} = -\frac{1}{i}$$

whence:

$$\frac{\partial C}{C \partial L} = -\frac{1}{i} \frac{\partial i}{\partial L}. \quad (70)$$

Thence for some abnormalities in dimensions, Hitchcock's formulae provide for determination of $\frac{\partial C}{C \partial L}$. There are additional abnormalities whose effects upon the permanent yaw may be discussed dynamically and the resultant effects on the elements X and Z found. In most remaining cases reliance must be placed either upon the results of resistance firings or upon firings for differential effects of normal and abnormal shell. An example of the latter type of firings and their reduction will be presented in the present report.

[1] H.P. Hitchcock: "A Study of Form Factors of Spinning Projectiles," Ballistic Research Laboratory Report No. 166.

It is now possible to return to the consideration of the effects of generalized abnormalities upon dispersion. It is noteworthy that the effects upon the variance $\Delta_L\sigma_X^2$ and $\Delta_L\sigma_Z^2$, arise from augmentation of the variance in ε_L: that is, it is supposed that unless the quantity $\sigma_L^{2\prime} - \sigma_L^2$ for the lot is zero, there is an effect upon the variance arising therefrom. The effects to be regarded as augmenting the variance in range and deflection are presumed to be due solely to the increase of the variance in ε_L. By equations (9):

$$\Delta_L\sigma_X^2 = \left(\frac{dX}{dL}\right)^2 \Delta\sigma_L^2 = \left(\frac{dX}{dL}\right)^2 (\sigma_L^{2\prime} - \sigma_L^{2\prime})$$

$$\Delta_L\sigma_Z^2 = \left(\frac{dZ}{dL}\right)^2 \Delta\sigma_L^2 = \left(\frac{dZ}{dL}\right)^2 (\sigma_L^{2\prime} - \sigma_L^2). \tag{71}$$

It may be true that the presence of a difference, $\bar{\varepsilon}_L^{\prime} - 0$, between the normal and abnormal population means in reality results in augmenting the dispersion in range and deflection even if $\sigma_L^{2\prime} = \sigma_L^2$. However, this augmentation is difficult to calculate from dynamical principles, and its significance must be established by experiment. It appears unlikely that this latter augmentation is considerable provided that the lot population mean, $\bar{\varepsilon}_L^{\prime}$, is kept small. The available data are not sufficiently complete to provide the necessary evidence for its existence at present. The total true effect on variance will accordingly be considered that given by equations (9). The magnitude of the effects on the population variances in range and deflection will be considered independent of the magnitude of the lot mean of the abnormalities, $\bar{\varepsilon}_L^{\prime}$, and proportional to their deviation from normal population variance. Let it be supposed that two indefinitely numerous groups of shell were fired under identical conditions in which $\bar{\varepsilon}_L^{\prime} = \bar{\varepsilon}_L$ for the first group and $\bar{\varepsilon}_L^{\prime\prime} = \bar{\varepsilon}_L + \Delta\varepsilon_L$ for the second group. It appears that $\bar{X}^{\prime} = \bar{X}$ and $\bar{Z}^{\prime} = \bar{Z}$ in the first case and $\bar{X}^{\prime\prime} = \bar{X} + \Delta_{\Delta\varepsilon_L}\bar{X}$ and $\bar{Z}^{\prime\prime} = \bar{Z} + \Delta_{\Delta\varepsilon_L}\bar{Z}$ in the second but $\sigma_X^{2\prime} = \sigma_X^{2\prime\prime} = \sigma_X^2$ and $\sigma_Z^{2\prime} = \sigma_Z^{2\prime\prime} = \sigma_Z^2$ in both cases. In other words the population mean points of impact are affected by a constant departure in a mean characteristic between two groups, but the population dispersion is not so affected. On the other hand, a random departure whose mean is zero for the two groups but whose population variance is not the same in the two

groups causes a change in the population dispersions in range and deflection but no change in the population mean range and deflection. It is now supposed that ε_L is a random variable whose population mean is zero, and whose magnitude varies from shell to shell due to imperfections of material and workmanship. In the group considered, which may be great or small, it is further supposed that the population variance in ε_L differs from the population variance σ_L^2 for normal shell. Let it be supposed, by analogy with the foregoing, that the elementary effects on range and deflection are built up from effects of ε_L upon angle of departure, initial velocity and ballistic coefficient in the case of range and from effects of ε_L upon side-jump and drift in the case of deflection. That is:

$$\left.\begin{aligned} \Delta_L X &= \Delta_{\phi_L} X + \Delta_{v_{o_L}} X + \Delta_{C_L} X \\ \Delta_L Z &= \Delta_{I_L} Z + \Delta_{\zeta_L} Z \, . \end{aligned}\right\} \quad (72)$$

It will be noted that the equations (72) differ from the second form of (67) only in the fact that some terms have been combined. These terms are:

$$\left.\begin{aligned} \Delta_{\phi_L} X &= \Delta_{J_L} X + \Delta_{\varphi_{J_L}} X \\ \Delta_{I_L} Z &= \Delta_{J_L} Z + \Delta_{\varphi_{J_L}} Z \, . \end{aligned}\right\} \quad (73)$$

This combination is made because the effects on magnitude and orientation of jump on emergence due to yaw due to clearance require a single treatment in the present case. The effects of the components in the vertical and slant planes of the jump, J, will be more convenient to deal with in the case of dispersion than the effects of magnitude and orientation of J. There are several possible somewhat incorrect assumptions concerning the independence of various subsidiary effects upon the elements that

can be constructed from the action of a single typical abnormality ε_L. The least obnoxious of these assumptions appears to be that of attributing independence to the various terms appearing on the right hand side of equations (72). Accordingly, by the law for combining independent errors:

$$\left.\begin{aligned} \sigma^2_X &= \sigma^2_{X_0} + \sigma^2_{\Delta_{\phi_L}} X + \sigma^2_{\Delta_{v_{0_L}}} X + \sigma^2_{\Delta_{c_L}} X \\ \sigma^2_Z &= \sigma^2_{Z_0} + \sigma^2_{\Delta_{I_L}} Z + \sigma^2_{\Delta_{\zeta_L}} Z \,. \end{aligned}\right\} \quad (74)$$

The quantities σ^2_X and σ^2_Z are generalized: no specific magnitude has yet been assigned to σ^2_L. The quantities with zero subscripts are population variances which arise from the action of random variables unassociated with design. Then $\sigma^2_{X_0}$ and $\sigma^2_{Z_0}$ are the population variances of shell whose every characteristic L is exactly equal to the population mean value λ. The quantities having effect subscripts are the variances of the effects of ε_L upon the elements, these latter effects having been assumed independent. By definition, the variances are the mean values of the squares of the effects, that is:

$$\left.\begin{aligned} &\sigma^2_X = \Sigma\, \overline{(\Delta X)^2}\,, \quad \sigma^2_{X_0} = \overline{(\Delta_0 X)^2}\,, \quad \sigma^2_{\Delta_{\phi_L}} X = \overline{(\Delta_{\phi_L} X)^2} \\ &\sigma^2_{\Delta_{v_{0_L}}} X = \overline{(\Delta_{v_{0_L}} X)^2}\,, \quad \sigma^2_{\Delta_{c_L}} X = \overline{(\Delta_{c_L} X)^2} \\ &\sigma^2_Z = \Sigma\, \overline{(\Delta Z)^2}\,, \quad \sigma^2_{Z_0} = \overline{(\Delta_0 Z)^2} \\ &\sigma^2_{\Delta_{I_L}} Z = \overline{(\Delta_{I_L} Z)^2}\,, \quad \sigma^2_{\Delta_{\zeta_L}} Z = \overline{(\Delta_{\zeta_L} Z)^2} \,. \end{aligned}\right\} \quad (75)$$

The assumption that the effect is proportional to the magnitude of the disturbing cause transforms (74) into the form:

$$\left.\begin{aligned} \sigma_X^2 &= \sigma_{X_0}^2 + \left(\frac{\partial X}{\partial \Phi}\right)^2 \sigma_{\Phi_L}^2 + \left(\frac{\partial X}{\partial v_0}\right)^2 \sigma_{v_{0_L}}^2 + \left(\frac{\partial X}{\partial C}\right)^2 \sigma_{C_L}^2 \\ \sigma_Z^2 &= \sigma_{Z_0}^2 + \left(\frac{\partial Z}{\partial I}\right)^2 \sigma_{I_L}^2 + \left(\frac{\partial Z}{\partial \zeta}\right)^2 \sigma_{\zeta_L}^2 . \end{aligned}\right\} \quad (76)$$

Evidently the variances with the subscripts Φ_L, v_{0_L}, C_L, I_L and ζ_L are the mean squares of the effects on Φ, v_0, C, I and ζ arising from the action of ε_L. That is:

$$\left.\begin{aligned} \sigma_{\Phi_L}^2 &= \overline{(\Delta_L\Phi)^2}, \quad \sigma_{v_{0_L}}^2 = \overline{(\Delta_L v_0)^2}, \quad \sigma_{C_L}^2 = \overline{(\Delta_L C)^2} \\ \sigma_{I_L}^2 &= \overline{(\Delta_L I)^2}, \quad \sigma_{\zeta_L}^2 = \overline{(\Delta_L \zeta)^2} . \end{aligned}\right\} \quad (77)$$

The proportionality of the subsidiary effects to the magnitude of the fundamental disturbing cause, ε_L, may be used, and (76) appears in the form:

$$\left.\begin{aligned} \sigma_X^2 &= \sigma_{X_0}^2 + \left[\left(\frac{\partial X}{\partial \Phi}\right)^2\left(\frac{\partial \Phi}{\partial L}\right)^2 + \left(\frac{\partial X}{\partial v_0}\right)^2\left(\frac{\partial v_0}{\partial L}\right)^2 + \left(\frac{\partial X}{\partial C}\right)^2\left(\frac{\partial C}{\partial L}\right)^2\right] \sigma_L^2 \\ \sigma_Z^2 &= \sigma_{Z_0}^2 + \left[\left(\frac{\partial Z}{\partial I}\right)^2\left(\frac{\partial I}{\partial L}\right)^2 + \left(\frac{\partial Z}{\partial \zeta}\right)^2\left(\frac{\partial \zeta}{\partial L}\right)^2\right] \sigma_L^2 . \end{aligned}\right\} \quad (78)$$

The variance σ_L^2 is clearly the mean square of the random departure ε_L.

Since equations (78) compose the variances due to various sources for any fixed population, similar equations may be written down for any particular normal or abnormal lot. That is:

$$\left.\begin{aligned} \sigma_X^{2\prime} &= \sigma_{X_o}^2 + \left[\left(\frac{\partial X}{\partial \phi}\right)^2 \left(\frac{\partial \phi}{\partial L}\right)^2 + \left(\frac{\partial X}{\partial v_o}\right)^2 \left(\frac{\partial v_o}{\partial L}\right)^2 + \left(\frac{\partial X}{\partial C}\right)^2 \left(\frac{\partial C}{\partial L}\right)^2 \right] \sigma_L^{2\prime} \\ \sigma_Z^{2\prime} &= \sigma_{Z_o}^2 + \left[\left(\frac{\partial Z}{\partial I}\right)^2 \left(\frac{\partial I}{\partial L}\right)^2 + \left(\frac{\partial Z}{\partial \zeta}\right)^2 \left(\frac{\partial \zeta}{\partial L}\right)^2 \right] \sigma_L^{2\prime} . \end{aligned}\right\} \quad (79)$$

Subtraction of equations (78) from (79) leads to

$$\left.\begin{aligned} \Delta_L \sigma_X^2 &= \left[\left(\frac{\partial X^2}{\partial \phi}\right)^2 \left(\frac{\partial \phi}{\partial L}\right)^2 + \left(\frac{\partial X^2}{\partial v_o}\right)^2 \left(\frac{\partial v_o}{\partial L}\right)^2 + \left(\frac{\partial X}{\partial C}\right)^2 \left(\frac{\partial C}{\partial L}\right)^2 \right] \Delta \sigma_L^2 \\ \Delta_L \sigma_Z^2 &= \left[\left(\frac{\partial Z}{\partial I}\right)^2 \left(\frac{\partial I}{\partial L}\right)^2 + \left(\frac{\partial Z}{\partial \zeta}\right)^2 \left(\frac{\partial \zeta}{\partial L}\right)^2 \right] \Delta \sigma_L^2 . \end{aligned}\right\} \quad (80)$$

The explicit form (80) differs somewhat from the extended form of (9): the difference is due to the lack of cross-product terms in (80). This is a consequence of the attribution of independence to the terms on the right hand of (72). The augmentation of the variance in ε_L, $\Delta\sigma_L^2$, is of course the difference between the abnormal and normal population variances:

$$\Delta \sigma_L^2 = \sigma_L^{2\prime} - \sigma_L^2 . \quad (81)$$

The equations (35) yield:

$$\frac{\partial X}{\partial \Phi} = \frac{\partial X}{\partial E}$$

$$\frac{\partial Z}{\partial I} = X \sec E .$$

It will be observed that:

$$\frac{\partial Z}{\partial \zeta} = 1$$

$$\frac{\partial X}{\partial C}\frac{\partial C}{\partial L} = \frac{C\partial X}{\partial C}\,\frac{\partial C}{C\partial L} \,.$$

Making these substitutions in (80) results in:

$$\left.\begin{aligned} \Delta_L \sigma_X^2 &= \left[\left(\frac{\partial X}{\partial E}\right)^2\left(\frac{\partial \phi}{\partial L}\right)^2 + \left(\frac{\partial X}{\partial v_o}\right)^2\left(\frac{\partial v_o}{\partial L}\right)^2 + \left(\frac{C\partial X}{\partial C}\right)^2\left(\frac{\partial C}{C\partial L}\right)^2\right]\Delta\sigma_L^2 \\ \Delta_L \sigma_Z^2 &= \left[(X \sec E)^2\left(\frac{\partial I}{\partial L}\right)^2 + \zeta^2\left(\frac{\partial \zeta}{\zeta\partial L}\right)^2\right]\Delta\sigma_L^2 \,. \end{aligned}\right\} \quad (82)$$

The quantities $\frac{\partial X}{\partial E}$, $\frac{\partial X}{\partial v_o}$, $\frac{C\partial X}{\partial C}$, X sec E or their equivalents are all obtainable from firing tables. Let the quantities:

$$\left.\begin{aligned} \left(\frac{\partial \phi}{\partial L}\right)^2 \sigma_L^2 &= \overline{(\Delta_L \varphi)^2} = \left(\frac{\partial \phi}{\partial L}\right)^2 \overline{\varepsilon_L^2} \\ \left(\frac{\partial I}{\partial L}\right)^2 \sigma_L^2 &= \overline{(\Delta_L I)^2} = \left(\frac{\partial I}{\partial L}\right)^2 \overline{\varepsilon_L^2} \end{aligned}\right\}^{1} \quad (83)$$

be considered briefly. By equations (29) and (31):

$$\left.\begin{aligned} \phi &= E + j + J\cos\varphi_J \\ I &= \quad i_1 + J\sin\varphi_J \end{aligned}\right\} \quad (84)$$

$$\left.\begin{aligned} \Delta_L\phi &= \cos\varphi_J\left(\frac{\partial J}{\partial L}\right)\varepsilon_L - J\sin\varphi_J\left(\frac{\partial \varphi_J}{\partial L}\right)\varepsilon_L = \left(\frac{\partial \phi}{\partial L}\right)\varepsilon_L \\ \Delta_L I &= \sin\varphi_J\left(\frac{\partial J}{\partial L}\right)\varepsilon_L + J\cos\varphi_J\left(\frac{\partial \varphi_J}{\partial L}\right)\varepsilon_L = \left(\frac{\partial I}{\partial L}\right)\varepsilon_L \,. \end{aligned}\right\} \quad (85)$$

[1] It should be noted that the quantity $\overline{\varepsilon_L^2}$ is the mean square and not the square of the mean of ε_L.

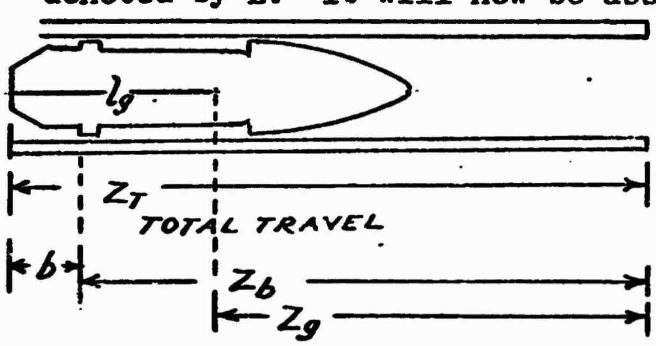
Let the expressions equated in (85) be squared and their means taken: the results are:

$$\left.\begin{aligned}
\overline{(\Delta_L\Phi)^2} &= \overline{\left[\cos^2\varphi_J\left(\frac{\partial J}{\partial L}\right)^2 - 2J\sin\varphi_J\cos\varphi_J\left(\frac{\partial\varphi_J}{\partial L}\right)\left(\frac{\partial J}{\partial L}\right) + J^2\sin^2\varphi_J\left(\frac{\partial\varphi_J}{\partial L}\right)^2\right]\varepsilon_L^2} \\
\overline{(\Delta_L I)^2} &= \overline{\left[\sin^2\varphi_J\left(\frac{\partial J}{\partial L}\right)^2 + 2J\sin\varphi_J\cos\varphi_J\left(\frac{\partial\varphi_J}{\partial L}\right)\left(\frac{\partial J}{\partial L}\right) + J^2\cos^2\varphi_J\left(\frac{\partial\varphi_J}{\partial L}\right)^2\right]\varepsilon_L^2}\,.
\end{aligned}\right\} \quad (86)$$

The averages (86) are taken with respect to the number of rounds over a great number of rounds. The last equation of (28) is:

$$\varphi_J = \frac{\pi}{2} + \psi_o + \frac{2\pi}{nd_H}(Z_g + l_g)\,.$$

It has been presumed that any dimension of the shell may be denoted by L. It will now be assumed that either L is independent of l_g and Z_g or that $L = l_g$.[1]



POSITION OF THE SHELL AT SEATING

The total travel of the base from the position of seating to the muzzle will be denoted by Z_T, and it will be supposed that the shell is so rammed that this distance is constant. Evidently

$$Z_T = l_g + Z_g\,.$$

It follows that

$$\varphi_J = \frac{\pi}{2} + \psi_o + \frac{2\pi}{nd_H}(Z_T)\,.$$

Thence

$$\frac{\partial\varphi_J}{\partial L} = 0\,.$$

In consequence:

$$\left.\begin{aligned}
\overline{(\Delta_L\Phi)^2} &= \overline{\left[\cos^2\varphi_J\left(\frac{\partial J}{\partial L}\right)^2\right]\varepsilon_L^2} \\
\overline{(\Delta_L I)^2} &= \overline{\left[\sin^2\varphi_J\left(\frac{\partial J}{\partial L}\right)^2\right]\varepsilon_L^2}\,.
\end{aligned}\right\} \quad (87)$$

Moreover $\left(\frac{\partial J}{\partial L}\right)$ is a constant independent of the number of rounds: in consequence (87) become:

$$\left.\begin{aligned}
\overline{(\Delta_L\Phi)^2} &= \left(\frac{\partial J}{\partial L}\right)^2\overline{\cos^2\varphi_J\,\varepsilon_L^2} \\
\overline{(\Delta_L I)^2} &= \left(\frac{\partial J}{\partial L}\right)^2\overline{\sin^2\varphi_J\,\varepsilon_L^2}\,.
\end{aligned}\right\} \quad (88)$$

[1] An analysis differing but slightly from the following could be made with the more general assumption that $L = p + q\,l_g$ where q is a pure number and p a general dimension independent of l_g.

Since φ_J is independent of ε_L, the mean with respect to the number of rounds of the product of a function of φ_J and a function of ε_L is the product of the separate means of the two functions with respect to the number of rounds. Hence:

$$\left.\begin{aligned}\overline{(\Delta_L\phi)^2} &= \left(\frac{\partial J}{\partial L}\right)^2 \overline{\cos^2\varphi_J}\ \overline{\varepsilon_L^2} = \overline{\cos^2\varphi_J}\left(\frac{\partial J}{\partial L}\right)^2 \sigma_L^2\\ \overline{(\Delta_L I)^2} &= \left(\frac{\partial J}{\partial L}\right)^2 \overline{\sin^2\varphi_J}\ \overline{\varepsilon_L^2} = \overline{\sin^2\varphi_J}\left(\frac{\partial J}{\partial L}\right)^2 \sigma_L^2.\end{aligned}\right\} \quad (89)$$

Then $\Delta_L J = \left(\frac{\partial J}{\partial L}\right)\sigma_L$ may be considered the radius of a circle. The probability that φ_J lies in the arc on this circle between φ_J and $\varphi_J + d\varphi_J$ is equal to $\frac{1}{2\pi} d\varphi_J$. The average with respect to the number of rounds of $\cos^2\varphi_J$ or $\sin^2\varphi_J$ is equal to the average of these functions with respect to φ_J around the circle. Then:

$$\left.\begin{aligned}\overline{\cos^2\varphi_J} &= \frac{\int_0^{2\pi}\cos^2\varphi_J d\varphi_J}{\int_0^{2\pi} d\varphi_J} = \frac{\frac{1}{2}2\pi}{2\pi} = \frac{1}{2}\\ \overline{\sin^2\varphi_J} &= \frac{1}{2}.\end{aligned}\right\} \quad (90)$$

Thence by (89), (90) and (83):

$$\overline{(\Delta_L\phi)^2} = \frac{1}{2}\left(\frac{\partial J}{\partial L}\right)^2 \sigma_L^2 = \left(\frac{\partial \phi}{\partial L}\right)^2 \sigma_L^2$$

$$\overline{(\Delta_L I)^2} = \frac{1}{2}\left(\frac{\partial J}{\partial L}\right)^2 \sigma_L^2 = \left(\frac{\partial I}{\partial L}\right)^2 \sigma_L^2$$

or

$$\left.\begin{aligned}(\frac{\partial\phi}{\partial L})^2 &= \frac{1}{2}(\frac{\partial J}{\partial L})^2\\ (\frac{\partial I}{\partial L})^2 &= \frac{1}{2}(\frac{\partial J}{\partial L})^2 .\end{aligned}\right\} \quad (91)$$

The equations (82) may now be written:

$$\left.\begin{aligned}\Delta_L\sigma_X^2 &= \left[\frac{1}{2}(\frac{\partial X}{\partial E})^2(\frac{\partial J}{\partial L})^2 + (\frac{\partial X}{\partial v_o})^2(\frac{\partial v_o}{\partial L})^2 + (\frac{C\partial X}{\partial C})^2(\frac{\partial C}{C\partial L})^2\right]\Delta\sigma_L^2\\ \Delta_L\sigma_Z^2 &= \left[\frac{1}{2}(X\sec E)^2(\frac{\partial J}{\partial L})^2 + \zeta^2(\frac{\partial\zeta}{\zeta\partial L})^2\right]\Delta\sigma_L^2 .\end{aligned}\right\} \quad (92)$$

The action of ε_L in changing $\mathcal{E}_c$ has been found to result in a velocity effect only, and this effect is:

$$\Delta_L v_o = (\frac{\partial v_o}{\partial L})\varepsilon_L = (\frac{\partial v_o}{\partial\mathcal{E}_c^2})(\frac{\partial\mathcal{E}_c^2}{\partial L})\varepsilon_L = \left\{-\frac{K_{D\delta}}{2}(\frac{2B}{A}-1)^2(\frac{S_o}{S_o-1})^{1/2}\right.$$

$$\left.\overline{Ev}\int_0^t(\frac{S_o v_o^2}{S_o v_o^2 - v^2})^{1/2} e^{-2h_1}\cos h^2(j_1-h_2)dt\right\}(\frac{\partial\mathcal{E}_c^2}{\partial L})\varepsilon_L ,$$

Consequently (92) become:

$$\left.\begin{aligned}\Delta_L\sigma_X^2 &= \left[\frac{1}{2}(\frac{\partial X}{\partial E})^2(\frac{\partial J}{\partial L})^2 + \left\{+\frac{K_{D\delta}\overline{Ev}}{2}(\frac{2B}{A}-1)^2\int_0^t(\frac{S_o v_o^2}{(S_o-1)(S_o v_o^2-v^2)})^{1/2}\right.\right.\\ &\qquad\left.\left. e^{-2h_1}\cos h^2(j_1-h_2)dt\right\}^2(\frac{\partial X}{\partial v_o})^2(\frac{\partial\mathcal{E}_c^2}{\partial L})^2 + (\frac{C\partial X}{\partial C})^2(\frac{\partial C}{C\partial L})^2\right]\Delta\sigma_L^2\\ \Delta_L\sigma_Z^2 &= \left[\frac{1}{2}(X\sec E)^2(\frac{\partial J}{\partial L})^2 + \zeta^2(\frac{\partial\zeta}{\zeta\partial L})^2\right]\Delta\sigma_L^2 ,\end{aligned}\right\} \quad (93)$$

Under the assumptions previously outlined $\frac{\partial J}{\partial L}$ and $\frac{\partial \varepsilon_C^2}{\partial L}$ may be obtained by differentiating partially, with respect to the particular L considered, the expressions:

$$\left.\begin{aligned} J &= \frac{\pi}{n}\left(\frac{d_H - d}{d_H}\right)\left(\frac{g - b}{l_1 + g - b}\right) \\ \varepsilon_C^2 &= \left(\frac{d_H - d}{2\left[l_1 + g - b\right]}\right)^2 . \end{aligned}\right\} \quad (94)$$

The coefficients $\left(\frac{\partial C}{C\partial L}\right)$ and $\left(\frac{\partial \varepsilon}{\varepsilon\partial L}\right)$ are obtained in the manner described in the discussion of the effects of ε_L on range and deflection. Accordingly the generalized effects on the variances in range and deflection due to the action of $\Delta\sigma_L^2$ may be considered completed. If there are more L's than one such that $\Delta\sigma_L^2$ is non-zero, the effects on the variances in range and deflection are additive.

4. Characteristic Abnormalities in Shell

Let certain characteristic abnormalities of shell be considered. These abnormalities have been defined as departures from normal population means in particular characteristics of shell. Four measurements, L, will be examined to some extent in the present report. Let:

- β denote the eccentricity of boattail;
- r denote the eccentricity of the center of mass;
- b denote the distance of the rear of the band from the base
- d denote the bourrelet diameter

The corresponding design values are $\beta_{\mathfrak{D}}$, $r_{\mathfrak{D}}$, $b_{\mathfrak{D}}$, and $d_{\mathfrak{D}}$. For any shell it is evident that:

$$\left.\begin{aligned} \beta_{\mathfrak{D}} &= 0 \\ r_{\mathfrak{D}} &= 0. \end{aligned}\right\} \quad (95)$$

The generally unknown population means referred to all manufactured shell for these characteristics are $\bar{\beta}$, $\bar{r}$, $\bar{b}$, and $\bar{d}$. It will be supposed that the sample of the shell employed in range firing for firing tables is sufficiently large that good estimates of both the means of L and the variances in L for individual rounds can be obtained. That is, it will be assumed that good estimates of

$\bar{\beta}$, $\bar{r}$, $\bar{b}$, $\bar{d}$, σ_β^2, σ_r^2, σ_b^2 and σ_d^2 are known. Then the departures from population means, ε_L, are approximated to sufficient accuracy for present purposes by:

$$\left.\begin{aligned}
\varepsilon_\beta &= \beta - \bar{\beta} = \beta - \beta_{\mathcal{D}} - (\bar{\beta} - \beta_{\mathcal{D}}). \\
\varepsilon_r &= r - \bar{r} = r - r_{\mathcal{D}} - (\bar{r} - r_{\mathcal{D}}) \\
\varepsilon_b &= b - \bar{b} = b - b_{\mathcal{D}} - (\bar{b} - b_{\mathcal{D}}) \\
\varepsilon_d &= d - \bar{d} = d - d_{\mathcal{D}} - (\bar{d} - d_{\mathcal{D}}).
\end{aligned}\right\} \quad (96)$$

It will appear that in some particular cases $L - L_{\mathcal{D}}$ is so large in comparison with $\lambda - L_{\mathcal{D}}$ that the latter is negligible in comparison therewith. Whenever this is true for any particular L, the required equation will be written down. In consequence, it is necessary to differentiate between normal and standard shell, the latter being shell all of whose dimensions have the design values $L_{\mathcal{D}}$. The abnormalities ε_L listed in (96) are by no means all those present in the shell dealt with in the present report, but the magnitudes of most of the other abnormalities were not measured.

In general, the estimates of λ and σ_L^2 are based upon a rather large, single sample of shell. This sample is of shell used in the range firing program for the firing table for 155mm. Howitzer, Shell Mk. I with M46 Fuze[1]. This sample has 200 rounds and therefore

$$\frac{n}{n-1} = 1.00502;$$

then $\frac{n}{n-1}$ may be taken as unity to three figure accuracy. Let t_L denote one half the total one-direction tolerances. The results are recorded in Table 1.

[1] FT-155-D-3

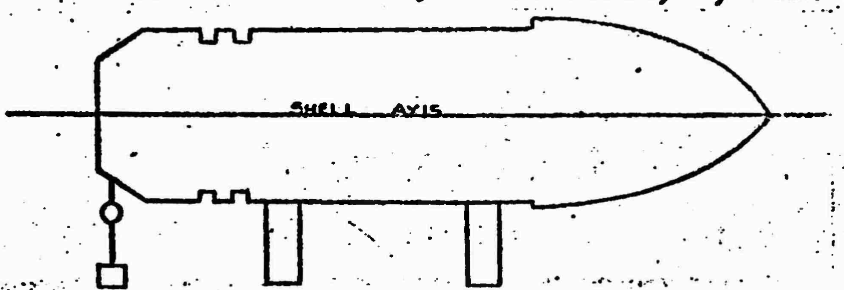
Table 1

L	$\bar{L}$	L_D	$\hat{s}^2_L$	t_L
β		0		not assigned
r		0		not assigned
b	3.1705 (IN.)	3.215 (IN.)	0.000141 (IN.^{2})	0.015 (IN.)
d	6.0781 (IN.)	6.080 (IN.)	0.000010 (IN.^{2})	0.005 (IN.)

A drawing of these shell is included herewith.

The nature of these characteristic dimensions and their abnormalities will be discussed further in order to provide for a clear treatment of their individual dynamical effects. The employment of tolerances in manufacture will be facilitated if the tolerances are given for quantities which are directly and readily measurable by ordinary machine gauges. The eccentricities to be governed by tolerances are accordingly defined.

The eccentricity in boattail, β, will be defined as one half the difference of maximum and minimum indicator readings one half inch from the shell base. The position of the indicator is shown in Fig. 10. If the indicator reading be denoted by ρ, the value of β is given by the formula:



MEASUREMENT OF ECCENTRICITY IN BOATTAIL

Fig. 10

$$\beta = \frac{1}{2}(\rho \text{ max.} - \rho \text{ min.}). \tag{97}$$

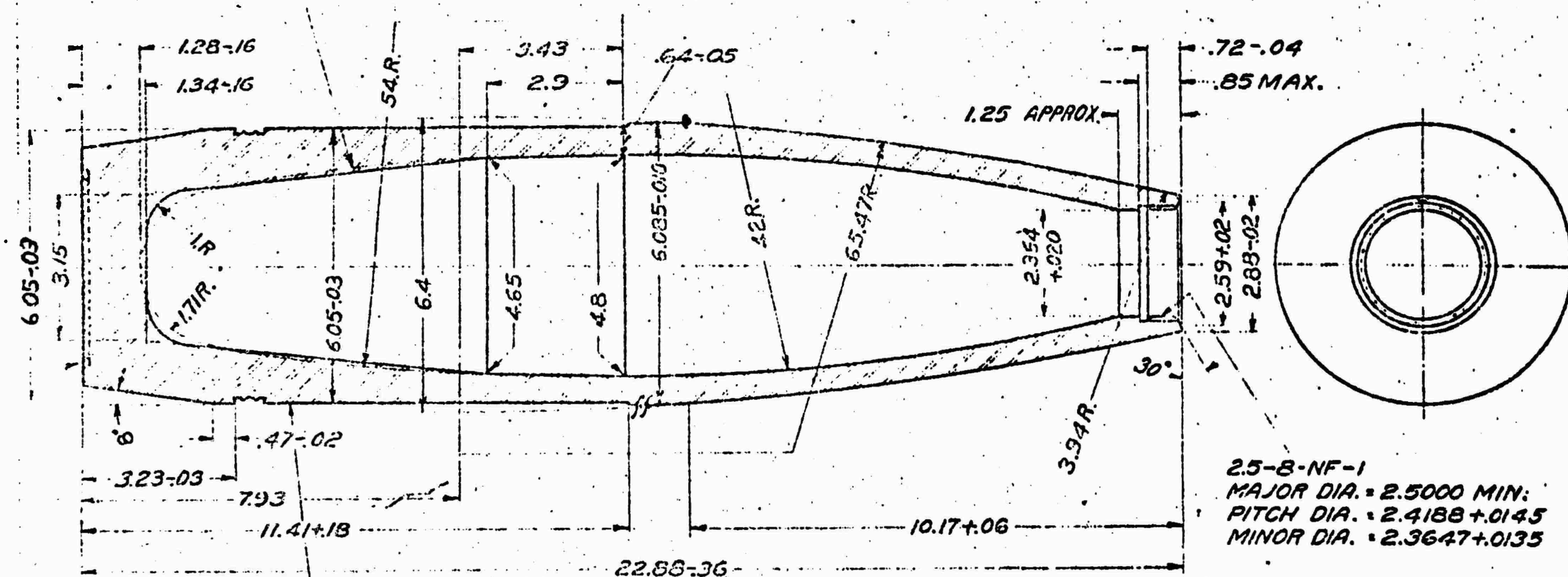
The value of β is given in inches.

The eccentricity of center of mass, r, denotes what is commonly described in vibration mechanics as static unbalance.[1] The eccentricity of center of mass is the distance from the center of mass of the shell to its longitudinal axis of figure. This quantity is evidently related to the eccentricity in mass distribution, (δ_m), which may be defined as the number of ounces necessary

[1] An interesting discussion of some dynamical effects resulting from both static and dynamic unbalance is contained in a report by H.P. Hitchcock: "Unbalance of 3" C.S. Shell 1915." It is presumed in the present report that the shell discussed are statically unbalanced without being dynamically unbalanced, that is the nearly longitudinal principal axis of inertia is presumed parallel to the axis of figure.

155 MM. SHELL MK. I FOR HOWITZER

(Abstract from Drawing 75-4-25, last revision 3-8-39)



NOTES:

A. Mean Volume of Capacity to Overflowing 272.3 cu.in.

Fig 9

Weights	
Part	Pounds
Band, Rotating	.76
Body, Shell	76.16
Cover, Base, complete	.53
Total Weight, empty	77.45 ±1.13
Plug, Lifting	2.25
Shipping Weight	79.70

to add to the light side of the shell at distance R from the axis of figure in order to make the longitudinal axis of mass distribution coincide with the longitudinal axis of figure. The relation between r and (δm) is given by the formula

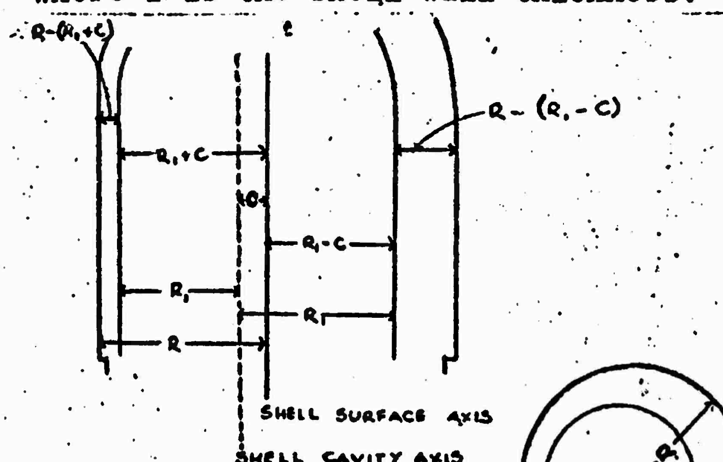
$$r \cdot m = (\delta m)R \tag{98}$$

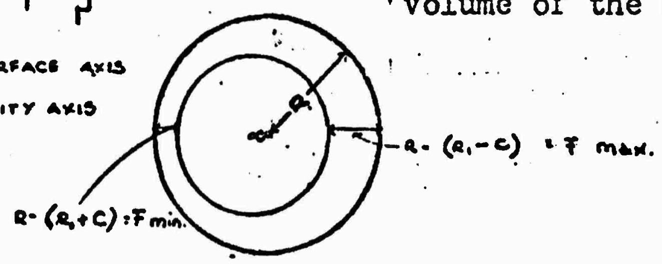
where R is the exterior radius of the shell between the band and the bourrelet. The eccentricity of mass distribution, (δm), may result from different causes. In case (δm) is due solely to the eccentricity in cavity, c, the latter may be computed from the former if the dimensions and masses of the different parts of the complete shell are given. The method may be reversed, and the procedure given below provides for computation of the eccentricity in mass when the eccentricity in cavity is given. The eccentricity in cavity, c, is given in inches and may be defined as the distance between the longitudinal axis of the shell surface and the longitudinal axis of the cavity. It may be more readily obtained in a machine shop by measuring the maximum and minimum shell wall thickness at a fixed distance from the shell base. Let R_1 denote the radius of the interior of the shell between the band and the bourrelet. The maximum thickness of the shell wall is $R - (R_1 - c)$ and the minimum thickness of the shell is $R - (R_1 + c)$. Then:

$$\left.\begin{aligned} c &= \frac{1}{2}\left[R - (R_1 - c) - R + (R_1 + c) \right] \\ &= \frac{1}{2}\left[\text{Ŧ}_{max.} - \text{Ŧ}_{min.} \right] \end{aligned}\right\} \tag{99}$$

where Ŧ is the shell wall thickness.

The method of computation of (δm) when c is known will be given, since this is quite likely to be of interest for shell other than those examined in the present report. It is necessary to calculate, or to be provided with, the volume of the cavity.



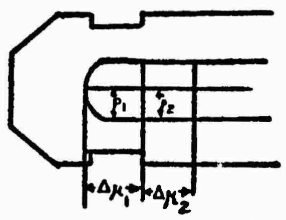


MEASUREMENT OF ECCENTRICITY IN CAVITY

Fig. 11

If μ is the distance of a point on the shell axis measured from the base of the shell, let μ be sectioned into short lengths, $\Delta\mu_i$, along the length of the cavity. The values of $\Delta\mu_i$ may be taken in the neighborhood of one inch for large shell. The radius of the cavity, P_i, should be taken at the midpoint of each $\Delta\mu_i$. The small finite volumes may be regarded as if replaced by cylindrical volumes if $\Delta\mu_i$ is chosen sufficiently small. The total volume of the shell cavity, v_c, is given approximately by:

$$v_c = \pi \sum_{i=1}^{n} P_i^2 \, \Delta\mu_i \qquad [\mathrm{IN.}^3] \qquad (100)$$



BREAKDOWN OF CAVITY VOLUME INTO ELEMENTARY VOLUMES

Fig.12

This computation may be avoided when the capacity of the shell with fuze inserted is given. This capacity is 265.5 $[\mathrm{IN}^3]$ for the 155mm. Howitzer H.E. Shell Mk.I. The computation may also be avoided when both the charge mass and density are given, except that the mass and density are ordinarily known to lower accuracy than the capacity as obtained by other methods. The mass of the cavity when filled with material of any density, ρ, can then be obtained by equation:

$$m = \rho v_c. \qquad (101)$$

It will be convenient for present purposes to employ the ounce per cubic inch as the unit of density. The density of many common materials varies in the third significant figure from sample to sample. The density of steel varies with carbon content and other factors and the density of the usual explosive loading is often more variable. The following rough figures will be employed for steel:

$$\rho_{st} = 490.0 \, [\mathrm{LB.FT.}^{-3}] = 4.537 \, [\mathrm{OZ.IN}^{-3}]. \qquad (102)$$

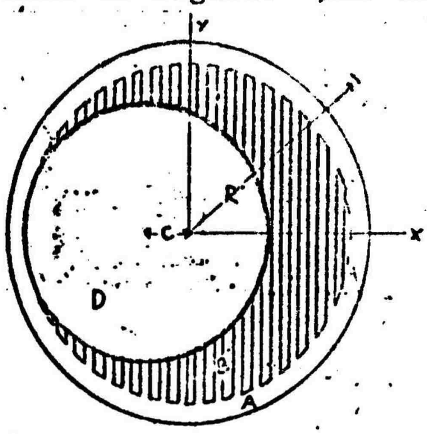
The resultant mass of the cavity of the 155mm. Howitzer H.E. Shell Mk.I when filled with steel, TNT and 50:50 Amatol is:

$$m_{c_{st}} = 1204.6 \ [OZ.]$$

$$m_{c_{TNT}} = 242.7 \ [OZ.] \qquad (103)$$

$$m_{c_{50:50\,Am}} = 227.2 \ [OZ.].$$

The computation of (δm) [OZ.], when c [IN.] is known, may now be considered. The fact that the shell has an exterior surface of revolution about the axis of figure will be useful for this purpose. R has been taken as the radius of the shell in inches between the band and the bourrelet. Consider a cross-section of the shell at any point, μ, along the axis of figure. The origin of coordinates x and y is on the axis of figure and the axes x and y lie in the cross-section. The x axis will be taken coincident with c and opposite in sense. Let $\bar{x}$ denote the x coordinate of the center of mass of any portion of the shell and m the corresponding mass. The subscripts, A, B and D will denote the three portions of the shell whose cross-sections at distance μ from the base are A, B and D. No subscripts will be employed for the complete shell.



CROSS-SECTION OF SHELL

Fig. 13

Evidently $\quad \bar{y} = \bar{y}_A = \bar{y}_B = \bar{y}_D = 0. \qquad (104)$

Taking moments about the axis of figure of the portions of the shell whose cross-sections at point μ along the axis of figure are A, B and D, it follows that:

$$r\ m = \bar{x}_A \cdot m_A + \bar{x}_B \cdot m_B + \bar{x}_D \cdot m_D. \tag{105}$$

It is clear from conditions of symmetry that:

$$\left.\begin{aligned} \bar{x}_A &= 0 \\ \bar{x}_D &= -c. \end{aligned}\right\} \tag{106}$$

Evidently the mass m_D is the mass of the charge, or filler:

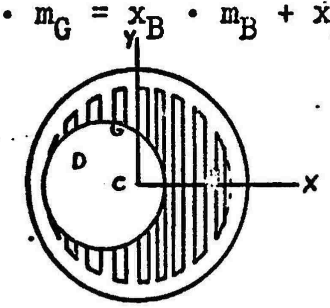
$$m_D = m_{c_{Fl}}. \tag{107}$$

Substitution of (106) and (107) into (105) leads to:

$$r\ m = \bar{x}_B \cdot m_B - cm_{c_{Fl}} \tag{108}$$

Let the portion of the shell whose cross-section is B be considered. If G is the whole cross-section area of B and D, and if the volume of which G = B + D is a cross-section were entirely filled with steel:

$$\bar{x}_G \cdot m_G = \bar{x}_B \cdot m_B + \bar{x}_D \cdot m_{c_{st}}. \tag{109}$$



SCHEME FOR DETERMINING AREAL MOMENTS OF SHELL CROSS SECTION

Fig.14

But if the colume of which G is a cross-sectio were filled with steel:

$$\bar{x}_G = 0. \tag{110}$$

Substituion of (110) and (106) into (109) leads to:

$$\bar{x}_B \cdot m_B = cm_{c_{st}}. \tag{111}$$

Substitution of (111) into (108) results in:

$$r\,m = c(m_{c_{ST}} - m_{c_{Fl}}) = c v_c(\rho_{ST} - \rho_{Fl}) \,. \tag{112}$$

Then the moment of the entire shell about the axis of figure is the product of the eccentricity in cavity and the difference in the mass of the cavity if filled with steel and the mass of the cavity with the usual explosive filler. Now (δm) is the mass which will balance the shell about the axis of figure if placed at the distance R from the axis of figure. Then:

$$r\,m - (\delta m)R = 0 \,. \tag{113}$$

Substitution of (113) into (112) leads to

$$(\delta m)R = c(m_{c_{ST}} - m_{c_{Fl}}) \,. \tag{114}$$

When the radius of the shell between the band and the bourrelet and the difference in mass of the cavity when filled with steel and with explosive filler are known, equation (114) provides for the calculation of (δm) from c.

Then:

$$\left.\begin{aligned} (\delta m) &= \frac{c(m_{c_{ST}} - m_{c_{Fl}})}{R} \\ c &= \frac{(\delta m)\,R}{(m_{c_{ST}} - m_{c_{Fl}})} \,. \end{aligned}\right\} \tag{115}$$

In the case of the 155mm. Howitzer H.E. Shell Mk.I:

$$\left.\begin{aligned} R &= 3.0175 \quad [IN.] \\ m_{c_{ST}} - m_{c_{TNT}} &= 1204.6 - 242.7 = 961.9 \quad [\;\;] \\ m_{c_{ST}} - m_{c_{50:50\,Am}} &= 1204.6 - 227.2 = 977.4 \quad [OZ.] \end{aligned}\right\} \tag{116}$$

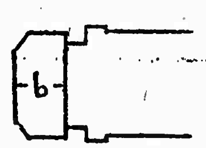
$$\left.\begin{aligned}
(\delta m)_{TNT} &= 318.8\ c \qquad [\mathrm{OZ.}]\\
(\delta m)_{50:50\ Am} &= 323.9\ c \qquad [\mathrm{OZ.}]\\
c &= 0.003137\ (\delta m)_{TNT} \qquad [\mathrm{IN.}]\\
c &= 0.003087\ (\delta m)_{50:50\ Am} \qquad [\mathrm{IN.}]\ .
\end{aligned}\right\} \qquad (117)$$

The measurement of (δm) must, if undertaken from the above, be carried out with shell containing the charge. The measurement must be taken at exactly R inches from the axis of figure, or corrected to that point. If the measurement is carried out with empty shell, $m_{c_{F1}} = 0$. For empty shell these results become:

$$\left.\begin{aligned}
(\delta m)_o &= 399.2\ c_o \qquad [\mathrm{OZ.}]\\
c_o &= 0.002505\ (\delta m)_o \qquad [\mathrm{IN.}].
\end{aligned}\right\} \qquad (118)$$

The 155mm. Gun H.E. Shell Mk.III has the same mass of charge of TNT as the 155mm. Howitzer H.E. Shell Mk.I. Comparison material from R. H. Kent's report referred to above will be employed for this shell. In general, this shell differs very slightly from the 155mm. Howitzer H.E. Shell Mk.I. The foregoing figures will be used for both types of shell without modification.

In the design of shell, the position of the band in respect to distance from the base, from the boattail and from the bourrelet is of importance. Suppose the length and angle of the boattail are constant. The distance of the band from the base is denoted by b. The band distance is indicated by Figure 15. If population mean value of b is $\bar{b}$, then:



BAND POSITION

Fig. 15

$$\varepsilon_b = b - \bar{b}. \qquad (119)$$

Other quantities are affected by a positive value of ε_b. If the angle of boattail,

length of boattail and diameter of the projectile behind the band are unchanged, the length of flat between the boattail and the band must be increased. If the flat length is F, then:

$$\varepsilon_F = F - \bar{F} = \varepsilon_b = b - \bar{b} . \quad (120)$$

The diameter of the bourrelet is also important. This quantity is denoted by d. By definition:

$$\varepsilon_d = d - \bar{d} . \quad (121)$$

5. Theoretical Effects of Characteristic Abnormalities of Shell.

In turning out boattails on shell, the axis of the boattail cone should be made to coincide with the axis of the shell within some small tolerance t_β. Lack of exact coincidence results in eccentricity of boattail, β. Some qualitative examination of the effects of this characteristic asymmetry may be made. In the first regime the action of the gas pressure in the gun upon an asymmetric boattail is likely to augment the yaw on emergence from the gun due to the yaw due to clearance, $\mathcal{E}_c$, unless $\mathcal{E}_c$ already attains θ_M without the additional action of β. It is probable that, in general, the existence of β to any considerable magnitude merely reduces the extremely small number of shell for which θ_M is not attained due to θ_0 having a value less than a few seconds of arc. In respect to all initial conditions for the second regime, therefore, it appears that there is no effect due to the action of β and in consequence, of ε_β, during the first regime.

Then:

$$\left.\begin{aligned}
&\Delta_{J_\beta} X = \Delta_{\varphi_{J_\beta}} X = \Delta_{v_{o_\beta}} X = 0 \\
&\Delta_{J_\beta} Z = \Delta_{\varphi_{J_\beta}} Z = 0 \\
&\Delta_{\varphi_\beta} \sigma_X^2 = \Delta_{v_{o_\beta}} \sigma_X^2 = 0 \\
&\Delta_{I_\beta} \sigma_Z^2 = 0 .
\end{aligned}\right\}$$

Substitution of the latter equations into (67) and (93) yields:

$$\left.\begin{aligned}
\Delta_\beta X &= \left(\frac{C\partial X}{\partial C}\right)\left(\frac{\partial C}{C\partial\beta}\right)\beta \\
\Delta_\beta Z &= \zeta\left(\frac{\partial\zeta}{\zeta\partial\beta}\right)\beta \\
\Delta_\beta \sigma_X^2 &= \left(\frac{C\partial X}{\partial C}\right)^2\left(\frac{\partial C}{C\partial\beta}\right)^2 \Delta\sigma_\beta^2 \\
\Delta_\beta \sigma_Z^2 &= \zeta^2\left(\frac{\partial\zeta}{\zeta\partial\beta}\right)^2 \Delta\sigma_\beta^2 .
\end{aligned}\right\} \quad (122)$$

It is necessary to provide the partial derivatives in order to furnish a computation form for later use. These are all zero except in respect to C and ζ, that is:

$$\left.\begin{aligned}
\frac{\partial J}{\partial\beta} &= 0 \\
\frac{\partial\varphi_J}{\partial\beta} &= 0 \\
\frac{\partial\varepsilon_c^2}{\partial\beta} &= 0 \\
\left(\frac{\partial J}{\partial\beta}\right)^2 &= 0 \\
\left(\frac{\partial\varepsilon_c^2}{\partial\beta}\right)^2 &= 0 .
\end{aligned}\right\} \quad (123)$$

Then the effects of β on range, deflection and dispersion result entirely from a change in the form of the equations of motion. This change may be treated as a change in the mean effective ballistic coefficient and the drift, or drift coefficient.

The eccentricity of boattail, β, results in a change in the equations of motion of a complicated character, and no attempt will be made to write down the exact form of the modified equations. In the first place, the eccentricity of boattail will result in the center of pressure departing from the axis of the projectile and lying permanently eccentric with respect to the

axis of figure. The magnitude of this departure is probably obtainable by experiment, but appears difficult to determine theoretically. Augmentation of the yaw will result and thereby an increase in the drag as well as an increase in the cross-wind force. The effects may be obtained by an appropriate modification of the accelerations due to drag and cross-wind force in the equations of motion. However the effects may be obtained equally well by changing the values of C and C_L in the following equations:[1]

$$\left.\begin{aligned} \frac{d\dot{x}}{dt} &= -\frac{GH}{C}\dot{x} \\ \frac{d\dot{y}}{dt} &= -\frac{GH}{C}\dot{y} - g \\ \frac{d\dot{z}}{dt} &= -\frac{GH\dot{z}}{C} + \frac{Q\dot{x}}{C_L}. \end{aligned}\right\} \quad (124)$$

The change in the yaw due to β appears difficult to determine theoretically. Therefore the quantities $\Delta_\beta C$ and $\Delta_\beta C_L$ must be obtained experimentally. The range and deflection will be modified appreciably if the asymmetry β is sufficiently great. If $\Delta\sigma_\beta^2$ is great enough, the dispersion will also be modified. It will be possible to use the equations (123) if $\left(\frac{\partial C}{C\,\partial\beta}\right)$ and $\left(\frac{\partial C_L}{C_L\,\partial\beta}\right)$ are determined experimentally. Since it is impossible to make shell with the boattail axis exactly coincident with the axis of the shell surface, it is desirable to establish a tolerance for the allowable eccentricity of boattail. Suppose that it is considered by artillerists that the resultant effect on range due to the action of β is the only basis for the employment of a tolerance, t_β. The artillerist may assign a limiting allowable range effect, $(\Delta_\beta X)_A$, by consideration of the accuracy desired for the range on the field of battle.

[1] These equations are equivalent to equations (78) of Dr. L. S. Dederick's Text on Ballistics published by the Ordnance School, Aberdeen Proving Ground, Md. The substitutions made in Dr. Dederick's equations in order to obtain equations (124) are those resulting from the assumption of no wind.

If $\left(\frac{\partial C}{C\partial\beta}\right)$ is known, the weight effect column of firing table may be used to compute $\left(\frac{dX}{d\beta}\right)$ by $\left(\frac{C\partial X}{\partial C}\right)\left(\frac{\partial C}{C\partial\beta}\right)$. The tolerance for β in manufacture may then be computed by:

$$t_\beta = \frac{(\Delta_\beta X)_A}{\left(\frac{dX}{d\beta}\right)} = \frac{(\Delta_\beta X)_A}{\left(\frac{C\partial X}{\partial C}\right)\left(\frac{\partial C}{C\partial\beta}\right)} = \frac{(\Delta_\beta X)_A}{c_\beta} \tag{125}$$

where c_β is $\frac{dX}{d\beta}$.

In turning out the cavity of shell, the axis of the cavity and of the fuze seating should be made to coincide with the axis of the exterior surface of the shell. Failure to achieve this coincidence results in the center of mass of the shell lying at some distance laterally from the axis of figure of the shell. Eccentricity of the cavity is one source of eccentricity of mass distribution. In the case of a nearly homogeneous steel shell, eccentricity of the cavity is by far the most important source of eccentricity of mass distribution. In order to balance these shell about the axis of figure, it would be necessary to add some weight to the light side of the shell.

It is noteworthy that eccentricity of the center of mass, r, will modify the motion of the axis of the shell inside the bore of the gun. The existence of $r > 0$ will, under some conditions, tend to augment the force producing the angular acceleration tending to drive the bourrelet toward the land-top. This acceleration is expressed by equation (11). In general θ very nearly attains θ_M without the additional action of r. The action of r inside the bore merely decreases the very small number of shell for which θ_M is not attained. This effect will be discounted. In respect to the jump on emergence due to the yaw due to clearance, however, the situation is decidedly different from the case of normal shell. Suppose, for simplicity, that the center of mass of the shell lies, throughout the motion of the shell inside the bore of the gun, in the same plane as the axis of the bore and the axis of the shell. Suppose further that the center of mass lies outside the acute angle formed by these two lines. Then, for normal shell, under the assumptions previously outlined, the jump on emergence due to the clearance inside the bore of the gun is given by one of the equivalent formulae:

$$\left.\begin{aligned}
J_\rho &= \theta_{\mathcal{M}} \frac{l_g}{v_o} \dot{\psi} = \theta_{\mathcal{M}} \left(\frac{2\pi l_g}{n d_H}\right) = \frac{\pi}{n}\left(\frac{d_H - d}{d_H}\right) \frac{l_g}{l_l} \\
\theta_{\mathcal{M}} &= \frac{d_H - d}{2 l_l} \\
\dot{\psi}_o &= \frac{2\pi}{n d_H} v_o \,.
\end{aligned}\right\} \quad (126)$$

In the case of shell with the eccentric center of mass lying in the plane of θ,[1] the quantity $\frac{r}{l_g}$ must be added to the value of $\theta_{\mathcal{M}}$ given by (126), in order to make the first equation of (126) hold. This follows from the fact that the center of mass for statically unbalanced shell lies at a distance , ρ, equal to $l_g\theta_{\mathcal{M}} + r$ from the axis of the bore on emergence. Then:[2]

$$\Delta_r J = \frac{r l_g}{l_g v_o} \dot{\psi} = \frac{r}{v_o}\left(\frac{2\pi}{n d_H} v_o\right) = \frac{2\pi}{n d_H} r \,. \quad (127)$$

Under the conditions previously outlined, the total values of the initial conditions at the muzzle are $\mathcal{E}_c^2$, $\varphi_{\mathcal{E}_c}$, J and φ_J.

Now these quantities are made up of two parts, the normal values and the perturbing effects of r. The normal values are distinguished by zero subscripts and the effects of r are denoted by the subscripts r. Then, by definition:

$$\left.\begin{aligned}
\mathcal{E}_c^2 &= \mathcal{E}_{c_o}^2 + \Delta_r \mathcal{E}_c^2 \\
\varphi_{\mathcal{E}_c} &= \varphi_{\mathcal{E}_{c_o}} + \Delta_r \varphi_{\mathcal{E}_c} \\
J &= J_o + \Delta_r J \\
\varphi_J &= \varphi_{J_o} + \Delta_r \varphi_J \,.
\end{aligned}\right\} \quad (128)$$

[1] Since the eccentric center of mass will not, in general, lie in the plane of θ, the results computed under this assumption are not exact. It may however, be presumed that the correct results would be obtained by multiplying those here given by a pure number with value between zero and unity.

[2] The augmentation due to eccentricity of the center of mass on the jump on emergence due to yaw due to clearance given by equation (127) was first given by H. P. Hitchcock in "Unbalance of 3" C.S. Shell, 1915."

The conditions underlying equations (10) are strictly geometrical. Therefore the perturbed quantities $\mathfrak{S}_c^2$ and $\varphi_{\mathfrak{S}_c}$ are known to be the same functions of θ, J, γ and ψ as the normal quantities are of θ_o, J_o, γ_o and ψ_o. By equations (10) and this consideration:

$$\left.\begin{aligned} \mathfrak{S}_c^2 &= \theta^2 + J^2 + 2\theta\, J \sin(\gamma-\psi) \\ \varphi_{\mathfrak{S}_c} &= \gamma - \text{arc sin}\left(\frac{\mathfrak{S}_c^2 + J^2 - \theta^2}{2\mathfrak{S}_c J}\right). \end{aligned}\right\} \qquad (129)$$

By (127), (128) and (129) it appears that:

$$\left.\begin{aligned} \Delta_r\mathfrak{S}_c^2 &= \mathfrak{S}_c^2 - \mathfrak{S}_{c_o}^2 = (\theta^2 - \theta_o^2) + (J^2 - J_o^2) + 2(\theta J \sin\left[\gamma - \psi\right] \\ &\quad - \theta_o J_o \sin\left[\gamma_o - \psi_o\right]) \\ \Delta_r\varphi\mathfrak{S}_c &= \varphi\mathfrak{S}_c - \varphi\mathfrak{S}_{c_o} = (\gamma - \gamma_o) - \left(\text{arc sin}\left[\frac{\mathfrak{S}_c^2 + J^2 - \theta^2}{2\mathfrak{S}_c J}\right]\right. \\ &\quad \left. - \text{arc sin}\left[\frac{\mathfrak{S}_{c_o}^2 + J_o^2 - \theta_o^2}{2\mathfrak{S}_{c_o} J_o}\right]\right) \\ \Delta_r J &= J - J_o = \frac{2\pi r}{nd_H} \\ \Delta_r\varphi_J &= \varphi_J - \varphi_{J_o}. \end{aligned}\right\} \qquad (130)$$

It has been remarked that, in the usual case, $\theta \longrightarrow \theta_M$ and $\theta \longrightarrow 0$ without the augmenting influence of r. Then $\Delta_r\theta$ is zero and θ very nearly attains θ_M. Making this substitution and using the third equation of (128) in the first equation of (130), the equations (130) become:

$$\left.\begin{aligned}
\Delta_r \mathfrak{C}_c^2 &= (2J_o \Delta_r J + \left[\Delta_r J\right]^2) + 2\theta_M\left(\left[J_o + \Delta_r J\right] \sin\left[\gamma - \Psi\right]\right. \\
&\quad \left. - J_o \sin\left[\gamma_o - \Psi_o\right]\right) \\
\Delta_r \varphi_{\mathfrak{C}_c} &= (\gamma - \gamma_o) - \left(\arcsin\left[\frac{\mathfrak{C}_c^2 + J^2 - \theta_M^2}{2\mathfrak{C}_c J}\right] - \arcsin\left[\frac{\mathfrak{C}_{c_o}^2 + J_o^2 - \theta_M^2}{2\mathfrak{C}_{c_o} J_o}\right]\right) \\
\Delta_r J &= \frac{2\pi r}{n d_H} \\
\Delta_r \varphi_J &= \varphi_J - \varphi_{J_o} .
\end{aligned}\right\} \quad (131)$$

Let ρ be the lateral distance of the center of mass of the shell from the axis of the bore at the instant of emergence of the band. The general definitions of φ_J and γ are given by:

$$\left.\begin{aligned}
\varphi_J &= \gamma + \frac{\pi}{2} \\
\gamma &= \arcsin\left(\frac{-\dot{\rho}\cos\Psi + \rho\dot{\Psi}\sin\Psi}{\sqrt{\dot{\rho}^2 + \rho^2\dot{\Psi}^2}}\right) .
\end{aligned}\right\} \quad (132)$$

In the case of shell with lateral displacement of the center of mass, r:

$$\left.\begin{aligned}
\dot{\rho} &= l_g \dot{\theta} \\
\rho &= (l_g \theta + r) .
\end{aligned}\right\} \quad (133)$$

By use of the conditions that $\dot{\theta} \longrightarrow 0$ and $\theta \longrightarrow \theta_M$, (133) may be transformed into:

$$\left.\begin{aligned}
\dot{\rho} &= 0 \\
\rho &= (l_g \theta_M + r) .
\end{aligned}\right\} \quad (134)$$

Substitution of (134) into (132) yields:

$$\left.\begin{aligned} \varphi_J &= \gamma + \frac{\pi}{2} \\ \gamma &= \text{arc sin}\left(\frac{(l_g\theta_M + r)\sin\Psi}{(l_g\theta_M + r)}\right) = \text{arc sin}(\sin\Psi) = \Psi. \end{aligned}\right\} \quad (135)$$

Substitution of (135) into (131) leads to:

$$\left.\begin{aligned} \Delta_r\mathfrak{C}_c^2 &= \Delta_r J(2J_o + \Delta_r J) \\ \Delta_r\varphi_{\mathfrak{C}_c} &= (\gamma - \gamma_o) - \left(\text{arc sin}\left[\frac{\mathfrak{C}_c^2 + J^2 - \theta_M^2}{2\mathfrak{C}_c J}\right] - \text{arc sin}\left[\frac{\mathfrak{C}_{c_o}^2 + J_o^2 - \theta_M^2}{2\mathfrak{C}_{c_o} J_o}\right]\right) \\ \Delta_r J &= \frac{2\pi r}{nd_H} \\ \Delta_r\varphi_J &= \gamma - \gamma_o = \Psi - \Psi_o \end{aligned}\right\} \quad (136)$$

Let (129) be employed in the quantities bracketed in the second equation of (136):

$$\left.\begin{aligned} \left[\frac{\mathfrak{C}_c^2 + J^2 - \theta_M^2}{2\mathfrak{C}_c J}\right] &= \frac{J + \theta_M \sin(\gamma - \Psi)}{\sqrt{\theta_M^2 + J^2 + 2\theta_M J \sin(\gamma - \Psi)}} \\ \left[\frac{\mathfrak{C}_{c_o}^2 + J_o^2 - \theta_M^2}{2\mathfrak{C}_{c_o} J_o}\right] &= \frac{J_o + \theta_M \sin(\gamma_o - \Psi_o)}{\sqrt{\theta_M^2 + J_o^2 + 2\theta_M J_o \sin(\gamma_o - \Psi_o)}} \end{aligned}\right\} \quad (137)$$

Employment of the second of (135) in (137) yields:

$$\left.\begin{aligned}\left[\frac{\mathfrak{C}_c^2 + J^2 - \theta_{\mathcal{M}}^2}{2\mathfrak{C}_c J}\right] &= \frac{J}{\sqrt{J^2 + \theta_{\mathcal{M}}^2}} \\ \left[\frac{\mathfrak{C}_{c_0}^2 + J_0^2 - \theta_{\mathcal{M}}^2}{2\mathfrak{C}_{c_0} J_0}\right] &= \frac{J_0}{\sqrt{J_0^2 + \theta_{\mathcal{M}}^2}}.\end{aligned}\right\} \quad (138)$$

Reduction of (136) with the third of (23) and (138) leads to:

$$\left.\begin{aligned}\Delta_r\mathfrak{C}_c^2 &= \left(\frac{2\pi r}{nd_H}\right)\left(2\theta_{\mathcal{M}}\left[\frac{2\pi l_g}{nd_H}\right] + \frac{2\pi r}{nd_H}\right) \\ \Delta_r\varphi_{\mathfrak{C}_c} &= (\Psi - \Psi_0) - \left(\text{arc sin}\left[\frac{J_0 + \Delta_r J}{\sqrt{J_0^2 + 2J_0\Delta_r J + [\Delta_r J]^2 + \theta_{\mathcal{M}}^2}}\right]\right. \\ &\quad \left. - \text{arc sin}\left[\frac{J_0}{\sqrt{J_0^2 + \theta_{\mathcal{M}}^2}}\right]\right) \\ \Delta_r J &= \frac{2\pi r}{nd_H} \\ \Delta_r\varphi_c &= \Psi - \Psi_0:\end{aligned}\right\} \quad (139)$$

For reasonably well-made shell, $r < l_g\theta_{\mathcal{M}}$ from which it follows that $\Delta_r J < J_0 < \theta_{\mathcal{M}}$, and that, without considerable error,

$$\text{arc sin}\frac{J_0 + \Delta_r J}{\sqrt{J_0^2 + 2J_0\Delta_r J + (\Delta_r J)^2 + \theta_{\mathcal{M}}^2}}$$

in the second equation of (139) may be replaced by

$$\frac{J_0 + \Delta_r J}{\sqrt{J_0^2 + 2J_0\Delta_r J + (\Delta_r J)^2 + \theta_{\mathcal{M}}^2}}$$

and $\text{arc sin}\dfrac{J_0}{\sqrt{J_0^2 + \theta_{\mathcal{M}}^2}}$ may be replaced by $\dfrac{J_0}{\sqrt{J_0^2 + \theta_{\mathcal{M}}^2}}$. Expansion

of the radicand in the expression $\dfrac{J_o + \Delta_r J}{\sqrt{J_o^2 + 2J_o\Delta_r J + (\Delta_r J)^2 + \theta_M^2}}$

by the binomial theorem and subtraction of the second expression in the parenthesis yields small terms. These terms will be neglected. The velocity in the bore and thereby the spin $\dot{\Psi}$ and the orientation Ψ have also a negligible effect due to r. Therefore (139) become:

$$\Delta_r \mathcal{E}_c^2 = \left(\frac{2\pi r}{nd_H}\right) \left(2\theta_M \left[\frac{2\pi l_g}{nd_H}\right] + \frac{2\pi r}{nd_H}\right)$$

$$\Delta_r \varphi \mathcal{E}_c \doteq 0$$

$$\Delta_r J = \frac{2\pi r}{nd_H}$$

$$\Delta_r \varphi_J \doteq 0 .$$

Since θ_M is unaffected by r, there can be no effect due to r on the extrapolation from $\mathfrak{E}_c$ to $\mathcal{E}_c$. Since the spin $\dot{\Psi}$ is unaffected by r, the extrapolation from $\varphi_{\mathfrak{E}_c}$ to $\varphi_{\mathcal{E}_c}$ is also unaffected by r. Since J and φ_J are constants with respect to time, no extrapolation is involved with respect to these quantities. Then:

$$\left.\begin{aligned}
\Delta_r \mathcal{E}_c^2 &= \left(\frac{2\pi r}{nd_H}\right)\left(2\theta_M \left[\frac{2\pi l_g}{nd_H}\right] + \frac{2\pi r}{nd_H}\right) \\
\Delta_r \varphi_{\mathcal{E}_c} &\doteq 0 \\
\Delta_r J &= \frac{2\pi r}{nd_H} \\
\Delta_r \varphi_J &\doteq 0
\end{aligned}\right\} \quad (140)$$

The equations (140) show a change in the initial conditions $\mathcal{E}_c^2$ and J with no concomitant changes in $\varphi_{\mathcal{E}_c}$ and φ_J. These results would be expected for shell with asymmetric mass distribution. It is noteworthy that these changes are entirely due to the change in the jump on emergence due to yaw due to clearance inside the bore of the gun. The two terms in $\Delta_r \mathcal{E}_c^2$ may be such that either the first or second is the larger. The quantity $\Delta_r \mathcal{E}_c^2$ is used in the equation:

$$\mathcal{E}_c^2 = \mathcal{E}_{c_o}^2 + \Delta_r \mathcal{E}_c^2 . \tag{141}$$

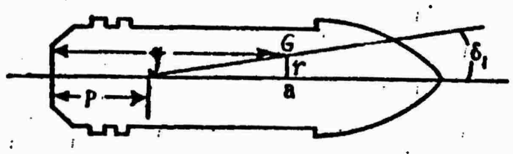
This is due to the fact that $\Delta_r \mathcal{E}_c^2$ is easier to formulate than $\Delta_r \mathcal{E}_c$ and further computations will not be impeded by its use. It is necessary to provide the partial derivatives needed to furnish the effects due to the action of r and $\Delta\sigma_r^2$ in changing the initial conditions at the muzzle of the gun for use in equations (67) and (93). Let the expressions (140) be divided[1] by r:

$$\left.\begin{aligned}
\frac{\partial J}{\partial r} &= \frac{2\pi}{nd_H} \\
\frac{\partial \varphi_J}{\partial r} &= 0 \\
\frac{\partial \mathcal{E}_c^2}{\partial r} &= \left(\frac{2\pi}{nd_H}\right)^2 \left(\left[\frac{d_H - d}{l_1}\right] l_g + r\right) \\
\left(\frac{\partial J}{\partial r}\right)^2 &= \left(\frac{2\pi}{nd_H}\right)^2 \\
\left(\frac{\partial \mathcal{E}_c^2}{\partial r}\right)^2 &= \left(\frac{2\pi}{nd_H}\right)^4 \left(\left[\frac{d_H - d}{l_1}\right] l_g + r\right)^2 .
\end{aligned}\right\} \tag{142}$$

After the shell leaves the gun, the eccentricity of the center of mass further modifies the character of the flight. The center of pressure will, for the present, be regarded as lying on the axis of the shell. The line between the center of pressure and the center of mass makes a small angle with the longitudinal axis of the shell. This angle will be denoted by δ_1. The tra-

[1] Division will yield correct results: it would be incorrect to differentiate $\mathcal{E}_c^2$ since this contains a significant term in r^2 as well as r. In most cases, the second order term in this equation may not be discounted.

jectory is, by definition, the path in space traversed by the center of mass, and angle of yaw is defined as the angle between the tangent to the trajectory and the longitudinal axis of the shell. Then the tangent to the trajectory passes through the center of mass of the shell and points in the direction of instantaneous motion of the center of mass. At the instant at which the center of mass-center of pressure line coincides with the tangent to the trajectory, the axis of figure departs therefrom by an angle, δ_1. Suppose that the spin of the projectile takes place about the center of mass-center of pressure axis[1], and let this spin be denoted by N in the usual manner. Let the point a be located on the perpendicular from the center of the mass to the axis of figure. Then the point a rotates about G with angular velocity N whatever be the motion of G. This constitutes an augmentation of the yaw whatever be the yaw. If no other yaw exists, the motion of the point a about G results in a yaw of constant amplitude. The amplitude of this yaw is given by:



CENTER OF PRESSURE AND CENTER OF MASS

Fig.16

$$\delta_1 = \frac{r}{g - p} . \tag{143}$$

The period of this yaw is:

$$P = \frac{2\pi}{N} . \tag{144}$$

Then, with respect to range, it appears that some addition to the drag due to yaw and some vertical displacement of the trajectory result from the effect of r upon the permanent yaw. The displacement in the vertical plane may probably be discounted but there should be an augmentation of the drift which is probably not negligible. It will then be necessary to determine $\frac{\partial C}{C\partial r}, \frac{\partial \zeta}{\zeta\partial r}$ experimentally for employment in equations (71) and (93). Thence the effects of asymmetry in center of mass and dispersion in the asymmetry of center of mass upon range, deflection and dispersion are given by:

[1] This assumption will not, in general, be satisfied. The result given is an example indicating that eccentricity of the center of mass tends to augment the permanent yaw. The result obtained will not be used for computation.

$$\Delta_r X=\left[\cos\varphi_J\left(\frac{2\pi}{nd_H}\right)\left(\frac{\partial X}{\partial E}\right)-\frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A}-1\right)^2\left\{\int_0^t\left(\frac{S_o^2v_o^2}{(S_o-1)(S_ov_o^2-v^2)}\right)^{\frac{1}{2}}e^{-2h_1}\cosh^2(j_1-h_2)\,dt\right.\right.$$

$$\left.\left(\frac{2\pi}{nd_H}\right)^2\left(\left\{\frac{d_H-d}{l_l}\right\}l_g+r\right)\frac{\partial X}{\partial v_o}+\left(\frac{C\partial X}{\partial C}\right)\left(\frac{\partial C}{C\partial r}\right)\right]r$$

$$\Delta_r Z=\left[\sin\varphi_J\left(\frac{2\pi}{nd_H}\right)X\sec E+\zeta\left(\frac{\partial\zeta}{\zeta\partial r}\right)\right]r$$

$$\Delta_r\sigma_X^2=\left[\frac{1}{2}\left(\frac{2\pi}{nd_H}\right)^2\left(\frac{\partial X}{\partial E}\right)^2+\left\{-\frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A}-1\right)^2\int_0^t\left(\frac{S_o^2v_o^2}{(S_o-1)(S_ov_o^2-v^2)}\right)^{\frac{1}{2}}e^{-2h_1}\cosh^2(j_1-h_2)\,dt\right\}^2\right.$$

$$\left.\left[\left(\frac{2\pi}{nd_H}\right)^4\left(\left\{\frac{d_H-d}{l_l}\right\}l_g+r\right)^2\right]\left(\frac{\partial X}{\partial v_o}\right)^2+\left(\frac{C\partial X}{\partial C}\right)^2\left(\frac{\partial C}{C\partial r}\right)^2\right]\Delta\sigma_r^2 \qquad (145)$$

$$\Delta_r\sigma_Z^2=\left[\frac{1}{2}\left(\frac{2\pi}{nd_H}\right)^2(X\sec E)^2+\zeta^2\left(\frac{\partial\zeta}{\zeta\partial r}\right)^2\right]\Delta\sigma_r^2 .$$

The effects given by (145) will be large if the quantities $r, \Delta\sigma_r^2$ are sufficiently great. Since it is impossible to make shell which are perfectly balanced statically, it is desirable to establish tolerances for eccentricity of center of mass. Let the bracketed coefficients denoted by total derivatives in (145) be supposed known and let the limiting allowable magnitude of the effect on range, $(\Delta_r X)_A$, be assigned by the using service as in the case of $(\Delta_\beta X)_A$. The tolerance, t_r, may be computed by:

$$t_r=\frac{(\Delta_r X)_A}{\left[\frac{dX}{dr}\right]}=\frac{(\Delta_r X)_A}{c_r} . \qquad (146)$$

A tolerance, $t_{\delta m}$, can be assigned immediately to actual eccentricity in mass by way of the equation (98). Then:

$$t_{\delta m} = t_r \frac{m}{R} = 502.1\ t_r \qquad [\mathrm{oz.}]. \qquad (147)$$

The band distance is of decided importance in respect to both aerodynamic efficiency and dispersion. The quantity $b_{\mathcal{D}}$ may be supposed coincident with $\bar{b}$ on an average over great numbers of shell. The effects upon range and deflection will be sought for ε_b, the departure from normal population mean band distance. In addition the effect on dispersion of $\Delta\sigma_b^2$ will be determined. The band position is measured from the base, but, physically, the distance from the base, from the boattail and from the bourrelet are all of importance.

The value of b exerts a slight modification of the motion inside the bore of the gun. The limiting value of the angle θ_M is affected slightly by a positive value of ε_b because of the effect of ε_b on l_1. In fact, the consequences on the initial conditions at the muzzle are obtainable from equations (28):

$$\left.\begin{aligned}
\Delta_b \mathcal{E}_c &= \frac{\partial \mathcal{E}_c}{\partial b}\,\varepsilon_b = + \frac{d_H - d}{2(l_1 + g - b)^2}\,\varepsilon_b = + \frac{\mathcal{E}_{c_o}}{l_1}\,\varepsilon_b \\
\Delta_b \varphi_{\mathcal{E}_c} &= \frac{\partial \varphi_{\mathcal{E}_c}}{\partial b}\,\varepsilon_b = + \frac{2\pi}{n d_H}\,\varepsilon_b \\
\Delta_b J &= - \frac{2\pi}{n d_H}\,\mathcal{E}_{c_o}\left[1 - \frac{l_g}{l_1}\right]\varepsilon_b \\
\Delta_b \varphi_J &= - \frac{2\pi}{n d_H}\,\varepsilon_b\,.
\end{aligned}\right\} \qquad (148)$$

Let the derivatives required for (67) and (93) be computed.

These are:

$$
\left.
\begin{aligned}
\frac{\partial J}{\partial b} &= -\frac{2\pi}{nd_H}\left[\frac{d_H - d}{2l_l^2}\right]\left[l_l - l_g\right] \\
\frac{\partial \varphi_J}{\partial b} &= -\frac{2\pi}{nd_H} \\
\frac{\partial \mathcal{E}_c^2}{\partial b} &= 2\mathcal{E}_c\frac{\partial \mathcal{E}_c}{\partial b} = +2\frac{\mathcal{E}_{c_o}^2}{l_l} = \frac{+2}{l_l}\cdot\left[\frac{d_H - d}{2l_l}\right]^2 \\
\frac{\partial \varphi_{\mathcal{E}_c}}{\partial b} &= +\frac{2\pi}{nd_H}.
\end{aligned}
\right\} \quad (149)
$$

The employment of (149) in (67) and (93) provides:

$$
\left.
\begin{aligned}
\Delta_b X = \Bigg[\Bigg\{ & \cos\varphi_J\left[-\frac{2\pi}{nd_H}\right]\left[\frac{d_H - d}{2l_l}\right]\left[\frac{l_l - l_g}{l_l}\right] + \sin\varphi_J\left[\frac{2\pi}{nd_H}\right]^2\left[\frac{d_H - d}{2l_l}\right]l_g\Bigg\}\frac{\partial X}{\partial E} \\
& - \frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A} - 1\right)^2\left\{\int_0^t\left(\frac{S_o^2 v_o^2}{(S_o - 1)(S_o v_o^2 - v^2)}\right)^{\frac{1}{2}} e^{-2h_1}\cos h^2(j_1 - h_2)\,dt\right\} \\
& \frac{2}{l_l}\left[\frac{d_H - d}{2l_l}\right]^2\frac{\partial X}{\partial v_o} + \frac{C\,\partial X}{\partial C}\frac{\partial C}{C\,\partial b}\Bigg]\varepsilon_b \\
\Delta_b Z = \Bigg[\Bigg\{ & \sin\varphi_J\left[-\frac{2\pi}{nd_H}\right]\left[\frac{d_H - d}{2l_l}\right]\left[\frac{l_l - l_g}{l_l}\right] - \cos\varphi_J\left[\frac{2\pi}{nd_H}\right]^2 \\
& \left[\frac{d_H - d}{2l_l}\right]l_g\Bigg\}(X \sec E)\Bigg]\varepsilon_b
\end{aligned}
\right\rangle \quad (150)
$$

$$\Delta_b \sigma_X^2 = \left[\frac{1}{2} \left\{ \left[\frac{2\pi}{n d_H} \right]^2 \left[\frac{d_H - d}{2 l_1} \right]^2 \left[\frac{l_1 - l_g}{l_1} \right]^2 \right\} \left(\frac{\partial X}{\partial E} \right)^2 \right.$$

$$+ \left\{ - \frac{K_{D_\delta} \overline{Ev}}{2} \left(\frac{2B}{A} - 1 \right)^2 \left[\int_0^t \left(\frac{\beta_o^2 \, v_o^2}{(S_o - 1) \; (S_o v_o^2 - v^2)} \right)^{\frac{1}{2}} e^{-2h_1 t} \cosh^2 (j_1 - h_2) \, dt \right] \right\}^2$$

$$\left. \left[\frac{2}{l_1} \left(\frac{d_H - d}{2 l_1} \right) \right]^2 \left(\frac{\partial X}{\partial v_o} \right)^2 + \left(\frac{C \partial X}{\partial C} \right)^2 \left(\frac{\partial C}{C \partial b} \right)^2 \right] \Delta \sigma_b^2 \qquad (150)\ \text{Cont'd.}$$

$$\Delta_b \sigma_Z^2 = \left[\frac{1}{2} \left\{ \left[- \frac{2\pi}{n d_H} \right]^2 \left[\frac{d_H - d}{2 l_1} \right]^2 \left[\frac{l_1 - l_g}{l_1} \right]^2 \right\} (X \sec E)^2 + \zeta^2 \left(\frac{\partial \zeta}{\zeta \partial b} \right)^2 \right] \Delta \sigma_b^2 ,$$

In addition to the effects of abnormality in band position on range, deflection and dispersion due to the effects of ε_b on the initial conditions ϕ, I, $\mathcal{E}_c$ and $\varphi_{\mathcal{E}_c}$, the abnormality in band position modifies the form of the equations of motion in air. This change in the form of the equations of motion consists in a change in the drag and a small change in the overturning moment coefficient. It is likely that the center of pressure is moved forward slightly when the band is moved forward and the center of mass will also be moved slightly forward. However, it is probable that the center of pressure is moved forward more than the center of mass. Then the arm of the moment of air force about the center of mass is increased. If other quantities remain the same, the moment coefficient will be increased and the yaw during the entire trajectory modified therefrom. Hence both an effect on the range and on the drift will ensue. The quantitative evolution of the latter effects must be found by experiment. The tolerance t_b, may be computed by analogy with formulae given before.

However, other dimensions are affected by a positive value of ε_b. If the angle of boattail, length of boattail and diameter of the projectile are unchanged, the length of the flat between the boattail and the band must be increased. Let the design flat length be denoted by F_D and the population mean flat length by $\overline{F}$. If the actual flat length, F and the other dimensions just referred to are unvaried, the length of flat between the boattail and the band must be increased. That is:

$$\varepsilon_F = F - \overline{F} = b - \overline{b} = \varepsilon_b . \qquad (151)$$

Up to some maximum, the increase in the length of flat will decrease the drag for zero yaw along the entire trajectory since the formation of eddies in the air is modified by the separation of sharp edges on the shell. If the yaw were unmodified, the result of this separation of sharp edges would be a positive range effect. Of course the action of ε_F will only change the initial conditions at the muzzle of the gun if $\varepsilon_F = \varepsilon_b$. Then, either the effects of ε_F and the effects of ε_b are not independent or the effects of ε_F and the effects of some other, possibly critical, dimension are not independent. The consequence is that unless $\varepsilon_F = \varepsilon_b$, the sign of the effects of ε_F can not be assigned in advance unless the dimension whose departure causes a concomitant departure in ε_F is known. In the shell dealt with in the present report, ε_F is not due to ε_b alone. It is necessary to consider that the effects of ε_F are dependent upon the effects of some other departures from population mean dimensions. The tolerance, t_F may be computed by analogy with formulae given before.

The diameter of the bourrelet is obviously of very great importance in determining the character of the initial conditions at the muzzle of the gun. The maximum value of θ, the angle between the axis of the shell and the axis of the bore is given by equation (27):

$$\theta_{\mathcal{M}} = \frac{d_H - d}{2\int_{\varsigma}}$$

By definition, $\varepsilon_d = d - \overline{d}$. It is easy to determine the partial derivatives necessary for equations (67) and (93) in respect to initial conditions. The results are:

$$\left.\begin{aligned}
\frac{\partial J}{\partial d} &= \frac{\partial}{\partial d}\left[\frac{d_H-d}{2l_l}\right]\left[\frac{2\pi l_g}{nd_H}\right] = -\frac{\pi}{nd_H}\left(\frac{l_g}{l_l}\right) \\
\frac{\partial \varphi_J}{\partial d} &= \frac{\partial}{\partial d}\left[\frac{\pi}{2} + \varphi_o + \frac{2\pi}{nd_H}\quad(Z_g + l_g)\right] = 0 \\
\frac{\partial \varepsilon_c^2}{\partial d} &= \frac{\partial}{\partial d}\left[\frac{d_H-d}{2l_l}\right]^2 = -\frac{1}{l_l}\left[\frac{d_H-d}{2l_l}\right] \\
\left(\frac{\partial J}{\partial d}\right)^2 &= \left[\left(\frac{\pi}{nd_H}\right)^2\left(\frac{l_g}{l_l}\right)^2\right] \\
\left(\frac{\partial \varepsilon_c^2}{\partial d}\right)^2 &= \left[\frac{1}{l_l^2}\left(\frac{d_H-d}{2l_l}\right)^2\right].
\end{aligned}\right\} \qquad (152)$$

It is notable that the effect of the departure from population mean diameter in changing the moments of inertia, the aero-dynamic coefficients and the quantities depending upon them all result in changes in the elements. Only one of these appears to have an effect which is resoluble in a simple manner. This effect will be deduced explicitly. All other effects of the departure from population mean bourrelet diameter will be assumed negligible. From the equation:

$$C = \frac{m}{id^2} \qquad (153)$$

it appears that:

$$\left.\begin{aligned}
\frac{\partial C}{C\partial d} &= -\frac{2}{d} \\
\frac{\partial \zeta}{\zeta\partial d} &= 0 .
\end{aligned}\right\} \qquad (154)$$

The employment of (152) and (154) in (67) and (93) yields:

$$\Delta_d X = \left[\left\{-\cos\varphi_J\left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right)\right\}\frac{\partial X}{\partial E}\right.$$

$$+\left\{\frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A}-1\right)^2\left[\int_0^t\left(\frac{S_o^2 v_o^2}{(S_o-1)\ (S_o v_o^2-v^2)}\right)^{\frac{1}{2}} e^{-2h_1}\cosh^2(j_1-h_2)\,dt\right]\right.$$

$$\left.\left[\frac{1}{l_l}\right]\left[\frac{d_h-d}{2\,l_l}\right]\right\}\frac{\partial X}{\partial v_o}-\left\{\frac{2}{d}\right\}\left.\frac{C\partial X}{\partial C}\right]\varepsilon_d$$

$$\Delta_d Z = \left[\left\{-\sin\varphi_J\left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right)\right\} X\sec E\right]\varepsilon_d$$

$$\Delta_d\sigma_X^2 = \left[\frac{1}{2}\left\{\left(\frac{\pi}{nd_H}\right)^2\left(\frac{l_g}{l_l}\right)^{2}\right\}\left(\frac{\partial X}{\partial E}\right)^2\right. \qquad (155)$$

$$+\left\{-\frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A}-1\right)^2\left[\int_0^t\left(\frac{S_o^2 v_o^2}{(S_o-1)\ (S_o v_o^2-v^2)}\right)^{\frac{1}{2}} e^{-2h_1}\cosh^2(j_1-h_2)\,dt\right]\right\}^2$$

$$\left.\left[\frac{1}{l_l}\right]^2\left[\frac{d_H-d}{2l_l}\right]^2\left(\frac{\partial X}{\partial v_o}\right)^2+\frac{4}{d^2}\left(\frac{C\partial X}{\partial C}\right)^2\right]\Delta\sigma_d^2$$

$$\Delta_d\sigma_Z^2 = \left[\frac{1}{2}\left\{\left(\frac{\pi}{nd_H}\right)^2\left(\frac{l_g}{l_l}\right)^2\right\}(X\sec E)^2\right]\Delta\sigma_d^2.$$

Equations (152) and (155) may be modified slightly and the results are:

$$\frac{\partial J}{\partial d} = -\left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right)$$

$$\frac{\partial \varphi_J}{\partial d} = 0$$

$$\frac{\partial \varepsilon_c^2}{\partial d} = -\left(\frac{1}{l_l}\right)\left(\frac{d_H - d}{2 l_l}\right)$$

$$\frac{\partial C}{C\partial d} = -\left(\frac{2}{d}\right)$$

$$\frac{\partial \zeta}{\zeta \partial d} = 0 \qquad (156)$$

$$\left(\frac{\partial J}{\partial d}\right)^2 = \left(\frac{\pi}{nd_H}\right)^2 \left(\frac{l_g}{l_l}\right)^2$$

$$\left(\frac{\partial \varepsilon_c^2}{\partial d}\right)^2 = \left(\frac{1}{l_l}\right)^2 \left(\frac{d_H - d}{2 l_l}\right)^2$$

$$\left(\frac{\partial C}{C\partial d}\right)^2 = \left(\frac{4}{d^2}\right)$$

$$\left(\frac{\partial \zeta}{\zeta \partial d}\right)^2 = 0 .$$

$$\Delta_d X = \left[-\cos\varphi_J \left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right)\left(\frac{\partial X}{\partial E}\right) + \frac{K_{D\delta}\overline{Ev}}{2}\left(\frac{2B}{A} - 1\right)^2 \left[\int_0^t \left(\frac{S_o^2 v_o^2}{(S_o - 1)\ (S_o v_o^2 - v^2)}\right)^{\frac{1}{2}} e^{-2h_1} \cosh^2(j_1 - h_2)\,dt\right]\left(\frac{d_H - d}{2 l_l^2}\right)\left(\frac{\partial X}{\partial v_o}\right) - \frac{2}{d}\left(\frac{C\partial X}{\partial C}\right)\right]\varepsilon_d$$

$$\Delta_d Z = \left[-\sin\varphi_J \left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right) X \sec E\right]\varepsilon_d$$

$$\Delta_d \sigma_X^2 = \left[\frac{1}{2}\left(\frac{\pi}{nd_H}\right)^2 \left(\frac{l_g}{l_l}\right)^2 \left(\frac{\partial X}{\partial E}\right)^2 + \left\{\left(\frac{K_{D\delta}\overline{Ev}}{2}\right)\left(\frac{2B}{A} - 1\right)^2 \int_0^t \left(\frac{S_o^2 v_o^2}{(S_o - 1)\ (S_o v_o^2 - v^2)}\right)^{\frac{1}{2}} e^{-2h_1} \cosh^2(j_1 - h_2)\,dt\right\}^2 \left(\frac{d_H - d}{2 l_l^2}\right)^2\left(\frac{\partial X}{\partial v_o}\right)^2 + \frac{4}{d^2}\left(C\frac{\partial X}{\partial C}\right)^2\right]\Delta\sigma_d^2 \qquad (157)$$

$$\Delta_d \sigma_Z^2 = \left[\frac{1}{2}\left(\frac{\pi}{nd_H}\right)^2 \left(\frac{l_g}{l_l}\right)^2 \left(X \sec E\right)^2\right]\Delta\sigma_d^2 .$$

The tolerance, ε_d, may be computed by formulae analogous to those given for the tolerances of other abnormalities.

(III) Measurements of Eccentricity in Shell.

1. Correspondence

The shell employed in these tests were encountered by the Savanna and Charleston Ordnance Depots. The correspondence relative to the sample of shell with eccentric boattails originated with Savanna Ordnance Depot. The significant portions are quoted in part.

O.O. 471.16/2245 (S.O.D. 471.1/488) Savanna Ordnance Depot

"1. In turning wide bands to narrow bands on 155mm. shell (loaded), this station has encountered a dozen or more which ran out badly in the lathe (shell are chucked at nose and tail). This run out was due to the axis of the boattail cone being at a noticeable angle to the axis of the shell body. One of these shell was measured up and the base found to run out 0.1 inch. In other words the center of the base was 0.05 inch away from the shell axis.

2. Instructions are requested as to whether shell such as those described are acceptable..."

1st Ind.

Ordnance Office

"1. It is requested that the 155mm. Shell, referred to in basic letter, and similar shell discovered in performing the band modification operation, be set aside.

2. After completion of the band modification program, the above shell will be chucked individually, the bands modified, and approximate measurements made to determine the eccentricity of the boattail.

3. For purposes of identification, it is desired that each of these shell be numbered, as it is expected that shipping orders will be furnished, directing them to the Aberdeen Proving Ground for use in connection with special test.

4. The number of shell held and the approximate eccentricity of each will be reported by indorsement hereon..."

2nd Ind.

Savanna Ordnance Depot

"1. 15 shell were found with boattails extremely eccentric. The eccentricity was measured by rolling the shell on their bodies on parallels laid on a surface plate, and taking indicator readings 1/2" from the base (most bases are slightly deformed at the edge so that it was impracticable to take readings at the edge of the base). The eccentricity was determined by halving the difference of the maximum and minimum indicator readings. The resulting figure represents the deviation of the axis of the boattail from the axis of the shell at a point 1/2" above the base... The eccentricities of the 15 shell are tabulated below.

Shell No.	Lot No.	Eccentricity of Boattail
1	5291-1	0.045"
2	-1	0.040
3	-1	0.047
4	-1	0.044
5	5291-4	0.030
6	-4	0.030
7	-4	0.043
8	-4	0.015
9	-4	0.030
10	5291-5	0.035
11	5291-26	0.023[1]
12	5291-27	0.020
13	-27	0.023
14	-27	0.020
15	-27	0.018

2. Shell No. 4 was found to be decidedly out of round. Actual measurements of the shell body at four points were: .000 - 0.030 - 0.007 - 0.033...."

3rd Ind.

Ordnance Office

"1. The above shell were ordered shipped to the Proving Ground for the purpose of determining the effect of eccentricity of boattail on range and dispersion..."

[1] This value is from the indicated letter. The result of a measurement by the Gauge Unit of the Aberdeen Proving Ground gave 0.033

4th Ind.

Aberdeen Proving Ground

"1. The fifteen 155mm. shell with eccentric boattails have been received. These shell are Type Mark I for the 155mm. Howitzer, ... M46 Fuze..."

5th Ind.

Ordnance Office

"1. The special firings outlined in paragraph 3 of the preceding indorsement are approved. They should, however, be held up until receipt of additional 155mm. Howitzer Shell from Charleston which are badly eccentric. In some cases such shell, when rolled on horizontal guides, require a weight as much as 34 ozs. to balance the heavy side. Samples of such shell with varying degrees of eccentricity are to be forwarded to the Aberdeen Proving Ground at an early date and it is believed that the firings can be combined utilizing the same group of standard shell...."

6th Ind.

Aberdeen Proving Ground

"1. Forwarded herewith is Firing Record No. 11004, covering the firings of 155mm. Howitzer Shell, eccentric in weight, and in axis of boattail, as directed in preceding correspondence.

2. Plot of ranges and deflections of these shell, as compared with 12 standard shell submitted for acceptance tests, is also enclosed. The additional sheet of the firing record gives the range of all shell, as corrected for weight of shell, and also the mean range of the various groups. It will be noted that the shell eccentric in boattail gave a mean range about 125 yards short of the standard shell, and those eccentric in weight gave a mean range about 200 yards short of standard. A brief study of the range - eccentricity data does not indicate a definitive relation between the amount of eccentricity and the range to be expected on individual shells.

3. Tabulation of the measurements of controlling dimensions and eccentricity of the shells are also enclosed...."

7th Ind.

Ordnance Office

"1. It is requested that the Ballistic Section make a study of the firing data to determine whether sufficient correlation of results can be determined

to justify fixing limits for rejection of projectiles in war reserve having either eccentric boattails or being eccentric as to weight.

2. Preliminary study undertaken in this office indicates that sufficient data on the various projectiles may not be available. Some of the data, including conditions, may be available at the Proving Ground.

3. It appears that the projectiles with eccentric boattails and with the greatest eccentricity as to weight were fired in order of the degree of variation from standard - that is, projectiles having the greatest variation were fired first. Examination of the results indicates that the earlier rounds fired were generally longer than the mean, and later rounds fell short. This would indicate that the projectiles having maximum variation from standard gave the greatest range. It is noted, however, that this same condition existed for the so-called "standard" projectiles - Lot 7884-1. It would therefore appear that some change occurred during the firing which progressively affected the range. The greatest degree of change appears to have occurred around noon. It is therefore requested that change in atmospheric conditions be investigated.

4. It is noted that the projectiles 1C to 15C, inclusive, showed a considerable difference in distance from rear of band to boattail— that is, difference in maximum and minimum on the same projectile. For example, Projectile No. 8C varied from 1.35 to 0.75, and Projectile 12C from 1.05 to 0.61. It is possible that this may have been due to eccentricity of the boattail in these projectiles, as well as eccentricity of weight. However, examination of the tabulated data in connection with Shell No. 1S to 15S, inclusive, indicates no correlation between eccentricity of boattail and variation (max. and min.) of distance from rear of band to boattail. As for example, projectile No. 15, which had eccentricity of boattail 0.045, had the distance from band to boattail varying only between 0.82 and 0.78 whereas projectile No. 9S, which had eccentricity of boattail 0.03, had the distance from rear of band to boattail varying between 1.02 and 0.65. The records do not indicate that any measurement was made of the eccentricity of boattail on other than the shell received from Savanna Ordnance Depot, or that eccentricity of weight was determined on projectiles other than those received from Charleston Ordnance Depot.

5. From measurements of distance "base to band", and distance "rear of band to boattail", it appears probable that projectiles, Lot 7884-1 were of the regular narrow band type, whereas the other lots were originally manufactured as broad

band and had the width of band reduced. The length of boattail varies slightly with these two types of projectile, which may partially account for the difference in mean range between the so-called "standard" projectiles and the other two types, although firings which were conducted some years ago did not indicate any appreciable difference in the outer zone of the 155mm. Howitzer.

6. Attention is invited to what appears to be a considerable number of typographical errors in the tabulation of "Dimensions of Shell Nos. 1S to 15S inclusive," diameters of shell body in rear of band, and in one case diameter of shell body, being given as 5 inches plus. It appears doubtful whether any of these dimensions could be less than 6 inches. . ." The questions discussed in the correspondence will be dealt with in detail in the following sections.

2. Results of Measurements.

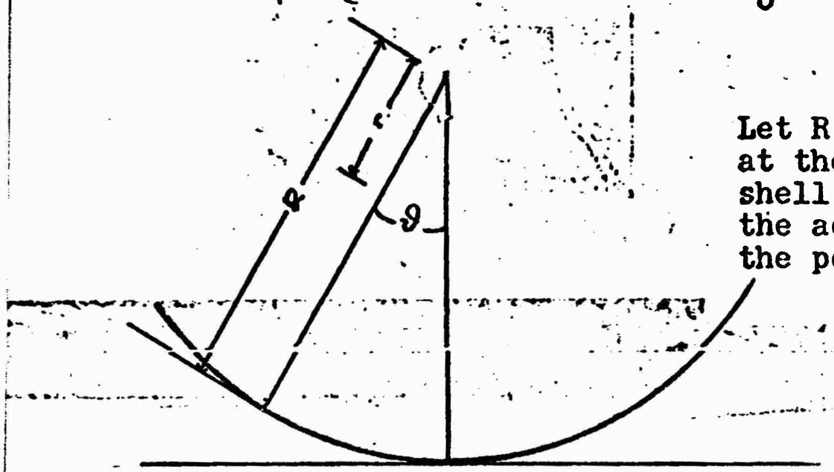
The Standard, Savanna (Eccentric Boattail) and Charleston (Eccentric Center of Mass) projectiles were all gauged by Mr. A. E. Jewell of the Gauge Unit, Aberdeen Proving Ground.[1] The boattail measurements made at Savanna are correct. The measurements of eccentricity of mass made at Charleston were determined by employment of a gauge of the type indicated in the discussion of eccentricity of center of mass.[2] It is not clear from the correspondence whether the centroid of the mass used to balance was at a distance precisely equal to R from the axis of figure. In general it would be desirable that the value of the mass to balance correspond to a distance which differs from R by less than 0.01 inch. It is possible to correct measurements of mass to balance made at any distance from the axis of figure to measurements corresponding to a distance equal to R. In order to make this correction it is necessary to know the dimensions of the gauge and the mass to be used.

[1] In regard to paragraph 6 of the 7th Indorsement of the correspondence quoted, a memorandum from Mr. Jewell to Major C. F. Hofstetter of the Proof Department dated June 3, 1938 may be quoted:

"The shell body diameters appearing as 5.0 are incorrect. An error of recording and not of measurement. . The boattail measurements are correct. . ."

[2] This gauge is described on page 512, "Manufacture of Artillery Ammunition."

A much better method of obtaining the eccentricity of the center of mass is commonly used at the proving ground.[1] This method consists in measuring the period of oscillation of the shell as it rolls on parallels about an equilibrium position. The eccentric line is that radius of the shell which passes through the center of mass. Let ϑ denote the angle between the eccentric line and the vertical through the axis of figure. Let ϑ_0 denote the maximum value of ϑ.



ROLLING OF THE SHELL
ON PARALLEL BARS
FIG. 17

Let R be the radius of the shell at the points of contact of the shell with the parallel bars, g the acceleration of gravity and T the period of one complete oscillation.

Further let

$$\left.\begin{aligned} k_1 &= \sin\left(\frac{\vartheta_0}{2}\right) \\ k_2 &= k_1\sqrt{\frac{4mr}{A+m(R-r)^2}} \\ K' &= \int_0^{\pi}\sqrt{\frac{1+k_2^2\sin^2\varphi}{1-k_1^2\sin^2\varphi}}\,d\varphi \end{aligned}\right\}\quad(158)$$

[1] This method is described by H. P. Hitchcock in "Eccentricity of 155 mm. Shell".

It is shown by Capt. T. E. Sterne[1] that

$$r = \frac{A + m(R - r)^2}{mg} \frac{16}{T^2} K'^2 .$$

Capt. Sterne's result may also be developed in the form[2]

$$r = \frac{A+mR^2}{mg} \frac{16}{T^2} K^2 \left\{1 - \frac{2r}{R} + \left(\frac{4mrR}{A+mR^2}\right) \left(\frac{K-E}{K}\right) + \dots \right\} .$$

The leading term of this development is given by H. P. Hitchcock in the report just cited. The values of ϑ_0 and T are observed. The remaining constants required for use with the shell discussed in the present report are[3]:

$$\left.\begin{aligned} A &= 3.349 \ [\text{LB.FT.}^{2}] \\ m &= 94.7 \ [\text{LBS.}] \\ R &= 0.2515 \ [\text{FT.}] \\ g &= 32.15 \ [\text{FT.SEC.}^{-2}] . \end{aligned}\right\} \quad (161)$$

The following short table presents the actual means of the samples:

	$\bar{\beta}$ [IN.]	$\overline{\delta m}$ [OZ.]	$\bar{b}$ [IN.]	$\bar{d}$ [IN.]	
Standard	—	—	3.222	6.0793	
Savanna (Eccentric boattail)	.0315	—	3.163	6.0778	(162)
Charleston (Eccentric mass)	—	32.43	3.175	6.0737 .	

[1] Note to H. P. Hitchcock, May 20, 1942. The derivation of Capt. Sterne's formula is preferable to one given by P. Kagan.

[2] K and E are the Complete Legendrian Elliptic Integrals of the First and Second Kind. These integrals are tabulated, for example, by B. O. Pierce in "A Short Table of Integrals."

[3] The mass and radius are for standard shell, that is, they are design values for 155mm.Howitzer H.E.Shell Mk.I with M46 fuze. The moment of inertia corresponds to a nearly similar shell the 155mm. Gun H.E. Shell, Mk.III with M46 fuze. This moment of inertia is an average value kindly given to the present writer by Mr. H. P. Hitchcock.

Although the quantities listed above are sample means they must be considered here as estimates of population means. It is unfortunate that no measurement of the small, but undoubtedly measurable, eccentricity of boattail in standard and eccentric mass projectiles was made. A similar lack of measurement of the eccentricity of mass in standard and eccentric mass projectiles is notable. The use of formulae (113) and (115) shows that:

$$\left.\begin{aligned} \bar{r} &= 0.06458 \ [\text{IN.}] \\ \bar{c} &= \underset{(\text{T.N.T.})}{0.10173 \ [\text{IN.}]} \ ; \ \underset{(50:50 \text{ Am.})}{0.10012 \ [\text{IN.}]} . \end{aligned}\right\} \qquad (163)$$

The sample variances $s^{2\,\prime}_{\beta}$ and $s^{2\,\prime}_{\delta m}$ may be taken as estimates of population variances: the $s^{2\,\prime}_{b}$ and $s^{2\,\prime}_{d}$ require correction by a factor of $\frac{n}{n-1}$ in order to obtain estimates of population variances. The estimates of population variances are given by $\hat{s}^2_t = \frac{n}{n-1} s^2$. These estimates are:

	$\hat{s}^2_{\beta}$ [IN.^{2}]	$\hat{s}^2_{\delta m}$ [OZ.^{2}]	$\hat{s}^2_{b}$ [IN.^{2}]	$\hat{s}^2_{d}$ [IN.^{2}]
Standard	—	—	0.000,275	0.000,001,89
Savanna (Eccentric boattail)	0.000,106	-	0.000,121	0.000,024,32
Charleston (Eccentric mass)	-	134.39	0.000,580,8	0.000,060,70

(164)

The departures of population means for the three samples from the population means of firing table shell are, of course, obtainable only by estimate. The estimates of population means from firing table shell are determined from the reduction of gauge measurements given in Appendix A. The reduction yields:

$$\text{Firing Table Shell Means } \bar{\beta} = - [\text{IN}], \bar{\delta m} = - [\text{OZ}], \bar{b} = 3.17 [\text{IN}],$$
$$\bar{d} = 6.078 [\text{IN}] \qquad (165)$$
$$\text{Firing Table Shell Variances } \hat{s}^2_{\beta} = - [\text{IN.}^2], \hat{s}^2_{\delta m} = - [\text{OZ.}^2], \hat{s}^2_{b} = 0.000,141 [\text{IN.}^2],$$
$$\hat{s}^2_{d} = 0.000,010,185 [\text{IN.}^2].$$

If the estimates of ε_L and $\Delta\sigma_L^2$ are used to replace the corresponding theoretical quantities, the differences in the sample population values and the firing table population values are:

	ε_β [IN.]	ε_r [IN.]	ε_b [IN.]	ε_d [IN.]	
Standard	—	—	+0.052	0.0013	
Savanna (Eccentric boattail)	+0.0315	—	-0.007	-0.0002	(166)
Charleston (Eccentric mass)	—	+0.064,58	+0.005	-0.0043.	

	$\Delta\sigma_\beta^2$ [IN.^{2}]	$\Delta\sigma_r^2$ [IN.^{2}]	$\Delta\sigma_b^2$ [IN.^{2}]	$\Delta\sigma_d^2$ [IN.^{2}]	
Standard	—	—	0.000,134	-0.000,008,3	
Savanna (Eccentric boattail)	0.000106	—	-0.000,020	+0.000,014,1	(167)
Charleston Eccentric mass)	—	+0.000,533	0.000,440	+0.000,050,5.	

3. Theoretical Effects of Observed Abnormalities of Shell.

The shape of these shell corresponds to the shape upon which the resistance firings for the J_5 resistance function are based. The current firing table for normal 155mm. Howitzer, H.E. Shell Mk. I with M46 fuze was issued February 1, 1940 and is numbered FT 155-D-3. The ballistic coefficient from the range firing reductions in the seventh zone for normal muzzle velocity 1478 [FT./SEC.]. at an angle of elevation 23° 15' was found to be 2.42. Three nearby mean points of impact for gauged shell with ten shell to each point assure the accuracy of the firing table values which are here employed.[1]

[1] These values were obtained from Mr. James Prevas and Mr. Verdon Atkins of the Ground Fire Unit of the Ballistic Research Laboratory.

The following constants will be used in the computations relating to these shell when fired under these conditions:

$$
\begin{aligned}
E &= 23^\circ 15' \\
v_o &= 1478 \ [\text{FT.SEC.}^{-1}] \quad \text{[1]} \\
T &= 30.1 \ [\text{SEC.}] \quad \text{[1]} \\
\overline{X} &= 9599 \ [\text{YDS.}] \quad \text{[2]} \\
\overline{Z} &= 101.8 \ [\text{YDS.}] \quad \text{[2]} \\
r_X^2 &= 2116 \ [\text{YDS.}^2] \quad \text{[2]} \\
r_Z^2 &= 36 \ [\text{YDS}^2] \quad \text{[2]} \\
\sigma_X^2 &= 4651 \ [\text{YDS.}^2] \\
\sigma_Z^2 &= 79 \ [\text{YDS.}^2] \\
\overline{j} &= 5.8 \ \cancel{m} \quad \text{[2]} \\
\frac{\partial X}{\partial E} &= 13.51 \ [\text{YDS.}\cancel{m}^{-1}] = 13765 \ [\text{YDS.RAD}^{-1}] \\
\frac{\partial X}{\partial v_o} &= +6.6 \ [\text{YDS. FT.}^{-1}\text{SEC.}] \quad \text{[2]} \\
\frac{\partial X}{\partial C} &= 2900 \ [\text{YDS.}] \quad \text{[2]} \\
X \sec E &= 10447? \ [\text{YDS.}] \quad \text{[2]} \\
\overline{\zeta} &= \overline{Z} = 101.8 \ [\text{YDS.}] \\
\overline{m} &= 94.7 \ [\text{LB.}] \quad \text{[2]} \\
\overline{B} &= 29.73 \ [\text{LB.FT}^{-2}] \quad \text{[3]} \\
\overline{A} &= 3.349 \ [\text{LB.FT}^{-2}] \quad \text{[3]} \\
\overline{A^2} &= 11.216 \ [\text{LB.}^2\text{FT}^{-4}]
\end{aligned}
\tag{168}
$$

1. Firing Record 11004
2. FT 155-D-3
3. H.P. Hitchcock: Measurements June, 1939
4. J. Prevas: Computations of FT 155-D-3

$$\left.\begin{array}{l}
\left\{\begin{array}{ll}
\bar{C} = 2.42 & 4 \\
\bar{K}_{D_\delta} = 0.005 \,[\mathrm{DEG.}^{-2}] & \\
\phantom{\bar{K}_{D_\delta}} = 16.41 \,[\mathrm{RAD.}^{-2}] & \\
\bar{K}_H = 3.058 \,[\mathrm{RAD}^{-2}] & 6 \\
\bar{K}_L = 0.821 & \\
\bar{K}_M = 1.22 & 3 \\
\bar{\rho}_{\infty} = 0.07513 \,[\mathrm{LB.FT.}^{-3}] & 5
\end{array}\right. \\
\left\{\begin{array}{l}
\bar{N}_o = \dfrac{2\pi}{n d_H} v_o = 713.8 \quad [\mathrm{RAD.SEC.}^{-1}] \\
\overline{\dfrac{AN}{B}} = 80.40 \\
\overline{\dfrac{2B}{A} - 1} = 16.76 \\
S_o = 1.8615 \\
\left(\dfrac{S_o}{S_o - 1}\right)^{1/2} = \left(\dfrac{S_o v_o^2}{S_o v_o^2 - v_o^2}\right)^{1/2} = 1.4700 \\
\left(\dfrac{S_o}{S_o - 1}\right) = 2.1608 \\
P_o = \left(\dfrac{S_o - 1}{S_o}\right)^{1/2} = 0.68029
\end{array}\right.
\end{array}\right\} \quad \begin{array}{l}(168)\\ \text{Cont'd.}\end{array}$$

3. H. P. Hitchcock: Measurements June, 1939
4. J. Prevas: Computations of FT 155-D-3
5. H. P. Hitchcock: Ballistic Research Laboratory Report 111:
C. "Aerodynamic Formulae and Nomenclature."
6. This value is assumed to be the same as Hitchcock's result with 3."3 CS. Shell fired at 2240 [FT./SEC.]

$$\left.\begin{array}{l}
\overline{n} = 25.586 \; ; \quad \overline{n^2} = 654.64 \\
\overline{d}_H = 6.102 \; [IN.] = 0.5085 \; [FT.] \\
\dfrac{2\pi}{nd_H} = 0.040,244 \; [RAD.IN^{-1}] = 0.48293 \; [RAD.FT.^{-1}] \\
\overline{Z}_o = \overline{Z_g + l_g} = 68.3857 \; [IN.] = 5.6988 \; [FT.] \\
Z_g = 62.2204 \; [IN.] = 5.1850 \; [FT.] \\
l_g = 6.1653 \; [IN.] = 0.5138 \; [FT.] \\
Z_g - l_g = 56.0551 \; [IN.] = 4.6713 \; [FT.] \\
\overline{\dfrac{2\pi}{nd_H}(Z_g - l_g)} = 2.255,9 \; [RAD.] \\
\overline{\dfrac{2\pi}{nd_H}(Z_g + l_g)} = 2.752,1 \; [RAD.] \\
\text{Total twist of rifling in howitzer} = 0.43802 \; [Turn] \\
= 2.752 \; [RAD.] = 157.7 \; [DEG.] = 5^h.26 \text{ o'clock.}
\end{array}\right\} \begin{array}{l}(168)\\ \text{Cont'd.}\end{array}$$

$$\left.\begin{array}{l}
\overline{d} = 6.0781 \; [IN.] = 0.506,50 \; [FT.] \; ; \; d^2 = 0.256,54 \; [FT.^2] \; ; \; d^4 \\
= 0.065,80 \; [FT.^4] \; d^5 = 0.033,32 \; [FT.^5] \\
\overline{g} = 1.536 \; [CAL.] = 9.3358 \; [IN.] = 0.777,98 \; [FT.]^{3} \\
\overline{b} = 3.1705 \; [IN.] \\
\overline{l}_1 = \overline{l}_g + \overline{l}_i = 8.28 \; [IN.] = 0.690 \; [FT.]
\end{array}\right\} (169)$$

[1] H. P. Hitchcock: Measurements June, 1939.

$$\left\{\begin{aligned} \bar{\varepsilon}_c &= \left(\overline{\frac{d_H - d}{2\ell_\ell}}\right) = 0.08268\ [\mathrm{DEG.}] = 0.001443\ [\mathrm{RAD.}] \\ \varepsilon_c^2 &= 0.006836\ [\mathrm{DEG.}^{+2}] \\ \left(\overline{\frac{2\pi \ell_g}{n d_H}}\right) &= 0.24812 \\ \bar{J} &= \bar{\varepsilon}_c \left(\overline{\frac{2\pi \ell_g}{n d_H}}\right) = 0.365\ [\not{M}] = 0.000358\ [\mathrm{RAD.}] \end{aligned}\right. \qquad (169)\ \text{Cont'd)}$$

$$\left\{\begin{aligned} &\left(\frac{\rho_\omega d^4 K_H}{B} + \frac{\rho_\omega d^2 K_L}{m}\right) = 0.000{,}675{,}7\ [\mathrm{FT}^{-1}]\ ; \ \frac{1}{2}\left(\frac{\rho_\omega d^4 K_H}{B} - \frac{\rho_\omega d^2 K_L}{m}\right) \\ &= 0.000{,}1708\ [\mathrm{FT.}^{-1}]\ . \end{aligned}\right.$$

The third regime is perhaps completely traversed within the first four seconds of the time of flight. The data relating to a normal trajectory during the first four seconds of the time of flight were obtained from the solution of the normal differential equations of motion by numerical integration in the standard manner. The results are given in the following short table:

155mm. Howitzer H.E. Shell Mk.I, M46 Fuze

$C = 2.42$ $\qquad\qquad$ $\overline{Ev} = 16.282\left[M\ SEC.^{-1}\right] = 53.419\left[FT.SEC.^{-1}\right]$

$\phi = E+j = 413.3 + 5.8\left[m\!\!\!/\right] = 419.1\left[m\!\!\!/\right]$

$v_o = 1478\left[FT.SEC.^{-1}\right]$

$$2h_1 = \int_o^t (f+k)dt$$

t	v	e^{-ay}	$S=S_o\left(\frac{v_o}{v}\right)^2$	$\left(e^{-ay}v\right)$	$\int_o^t e^{-ay}vdt$	$=\left(\frac{\rho_\omega d^4K_H}{B}+\frac{\rho_\omega d^2K_L}{m}\right)\int_o^t e^{-ay}vdt$
[SEC.]	[FT.SEC.$^{-1}$]	[1]	[1]	[FT.SEC.$^{-1}$]	[FT.]	[1]
0.0	1478.0	1.0000	1.8615	1478.0	0.0	0.0000
1.0	1390.7	0.9827	2.1025	1366.6	1420.5	0.9598
2.0	1318.8	0.9675	2.3381	1275.9	2740.1	1.8515
3.0	1262.3	0.9540	2.5520	1204.2	3978.6	2.6883
4.0	1219.5	0.9421	2.7343	1148.9	5154.0	3.4826

t	$\left(\frac{S}{S-1}\right)^{1/2}$	$\left(\frac{S}{S-1}\right)^{1/2}e^{-ay}v$	$\int_o^t\left(\frac{S}{S-1}\right)^{\frac{1}{2}}e^{-ay}vdt$
[SEC.]	[1]	[FT.SEC.$^{-1}$]	[FT.]
0.0	1.470	2172.7	0.0
1.0	1.381	1887.3	2021.7
2.0	1.322	1686.7	3802.9
3.0	1.282	1543.8	5414.4
4.0	1.256	1443.0	6904.5

(170)

$$h_2 = 1/2\int_o^t (f-k)\left(\frac{S}{S-1}\right)^{1/2}dt$$

t	$= 1/2\left(\frac{\rho_\omega d^4K_H}{B}-\frac{\rho_\omega d^2K_L}{m}\right)\int_o^t\left(\frac{S}{S-1}\right)^{1/2}e^{-ay}vdt$	j_1-h_2	e^{-2h_1}
[SEC.]	[1]	[1]	[1]
0.0	0.0000	0.0406	1.00000
1.0	0.3453	-0.3047	0.38297
2.0	0.6495	-0.6089	0.15700
3.0	0.9248	-0.8842	0.06800
4.0	1.1793	-1.1387	0.03073

t	$\cosh(j_1-h_2)$	$\cosh^2(j_1-h_2)$	$\left(\frac{\beta_o^2 v_o^2}{(\beta_o-1)(\beta_o v_o^2-v^2)}\right)^{1/2}$
[SEC]	[1]	[1]	[1]
0.0	1.0008	1.0016	2.1608
1.0	1.0467	1.0956	2.0299
2.0	1.1912	1.4190	1.9431
3.0	1.4170	2.0079	1.8849
4.0	1.7215	2.9636	1.8457

t	$\left(\frac{\beta_o^2 v_o^2}{(\beta_o-1)(\beta_o v_o^2-v^2)}\right)^{1/2} e^{-2h_1}\cosh^2(j_1-h_2)$
[SEC]	[1]
0.0	2.1642
1.0	0.8517
2.0	0.4329
3.0	0.2574
4.0	0.1681

170 cont'd

$$\left.\begin{array}{ccc} & \displaystyle\int_0^t \left(\frac{S_o{}^2 v_o{}^2}{(S_o-1)\,(S_o v_o-v^2)}\right)^{\frac{1}{2}} e^{-2h_1} \frac{K_{D_\delta}\overline{Ev}}{2}\left(\frac{2B}{A}-1\right)^2 \left(\frac{\partial X}{\partial v_o}\right) & \\ & \cosh^2(j_1-h_2)\,dt \qquad \displaystyle\int_0^t \left(\frac{S_o{}^2 v_o{}^2}{(S_o-1)(S_o v_o{}^2-v^2)}\right)^{\frac{1}{2}} e^{-2h_1} & \\ & \cosh^2(j_1-h_2)\,dt & \end{array}\right\}$$

t [SEC.]	[SEC.]	[YDS. RAD.$^{-2}$]
0.0	0.00	0.
1.0	1.40	$1.137(10^6)$
2.0	2.00	$1.625(10^6)$
3.0	2.35	$1.909(10^6)$
4.0	2.54	$2.063(10^6)$.

(171)

It will be necessary to assume the result of a frequent observation of R. H. Kent. It is usually true that shell are rammed home in such a way that their initial orientation at seating is zero: that is the bourrelet usually touches the uppermost land top on ramming.[1] Employing values taken from (169) and (171), the explicit values of (67) and (93) for the present shell are:

$$\Delta_L X = \left[-5223\,\frac{\partial J}{\partial L} + 4.56\,\frac{\partial \varphi_J}{\partial L} - 2.063(10^6)\,\frac{\partial \varepsilon_c{}^2}{\partial L} + 2900\,\frac{\partial C}{C\partial L}\right]\varepsilon_L$$

$$\Delta_L Z = \left[-9665\,\frac{\partial J}{\partial L} - 1.42\,\frac{\partial \varphi_J}{\partial L} + 101.8\,\frac{\partial \zeta}{\zeta\partial L}\right]\varepsilon_L$$

$$\Delta_L \sigma_X{}^2 = \left[9.474(10^7)\left(\frac{\partial J}{\partial L}\right)^2 + 4.256(10^{12})\left(\frac{\partial \varepsilon_c{}^2}{\partial L}\right)^2 + 3.410(10^6)\left(\frac{\partial C}{C\partial L}\right)^2\right]\Delta\sigma_L{}^2 \qquad (172)$$

$$\Delta_L \sigma_Z{}^2 = \left[5.457(10^7)\left(\frac{\partial J}{\partial L}\right)^2 + 10363\left(\frac{\partial \zeta}{\zeta\partial L}\right)^2\right]\Delta\sigma_L{}^2 \,.$$

[1] This undesirable assumption would have been rendered unnecessary if the orientation of the shell on ramming had been observed during range firing.

The quantities $\left(\frac{\partial J}{\partial L}\right)$, $\left(\frac{\partial \varphi_J}{\partial L}\right)$, $\left(\frac{\partial \mathcal{E}_c^2}{\partial L}\right)$, $\left(\frac{\partial C}{C\,\partial L}\right)$, and $\left(\frac{\partial \zeta}{\zeta\partial L}\right)$ have in part been evaluated for particular ε_{L_1} in Section II, 5 of the present report. It appears that:

$$\left.\begin{aligned}
&\left(\frac{\partial J}{\partial \beta}\right) = 0, \left(\frac{\partial \varphi_J}{\partial \beta}\right) = 0, \left(\frac{\partial \mathcal{E}_c^2}{\partial \beta}\right) = 0, \left(\frac{\partial J}{\partial \beta}\right)^2 = 0, \left(\frac{\partial \mathcal{E}_c^2}{\partial \beta}\right)^2 = 0 \\
&\left(\frac{\partial J}{\partial r}\right) = \frac{2\pi}{nd_H}, \left(\frac{\partial \varphi_J}{\partial r}\right) = 0, \left(\frac{\partial \mathcal{E}_c^2}{\partial r}\right) = \left(\frac{2\pi}{nd_H}\right)^2 \left(\left[\frac{d_H - d}{\ell_1}\right] \ell_g + r\right), \\
&\left(\frac{\partial J}{\partial r}\right)^2 = \left(\frac{2\pi}{nd_H}\right)^2, \left(\frac{\partial \mathcal{E}_c^2}{\partial r}\right)^2 = \left(\frac{2\pi}{nd_H}\right) 4 \left(\left[\frac{d_H - d}{\ell_1}\right] \ell_g + r\right)^2 \\
&\left(\frac{\partial J}{\partial b}\right) = -\left(\frac{2\pi}{nd_H}\right)\left(\frac{d_H - d}{2\ell_1}\right)\left(\frac{\ell_1 - \ell_g}{\ell_1}\right), \left(\frac{\partial \varphi_J}{\partial b}\right) = -\left(\frac{2\pi}{nd_H}\right) \\
&\left(\frac{\partial \mathcal{E}_c^2}{\partial b}\right) = +\frac{2}{\ell_1}\left(\frac{d_H - d}{2\ell_1}\right)^2 \\
&\left(\frac{\partial J}{\partial b}\right)^2 = \left(\frac{2\pi}{nd_H}\right)^2 \left(\frac{d_H - d}{2\ell_1}\right)^2 \left(\frac{\ell_1 - \ell_g}{\ell_1}\right)^2, \left(\frac{\partial \mathcal{E}_c^2}{\partial b}\right)^2 = \frac{4}{\ell_1^2}\left(\frac{d_H - d}{2\ell_1}\right)^4 \\
&\left(\frac{\partial J}{\partial d}\right) = -\left(\frac{\pi}{nd_H}\right)\left(\frac{\ell_g}{\ell_1}\right), \left(\frac{\partial \varphi_J}{\partial d}\right) = 0, \left(\frac{\partial \mathcal{E}_c^2}{\partial d}\right) = -\left(\frac{1}{\ell_1}\right)\left(\frac{d_H - d}{2\ell_1}\right) \\
&\left(\frac{\partial C}{C\partial d}\right) = -\left(\frac{2}{d}\right), \left(\frac{\partial \zeta}{\zeta\partial d}\right) = 0 \\
&\left(\frac{\partial J}{\partial d}\right)^2 = \left(\frac{\pi}{nd_H}\right)^2 \left(\frac{\ell_g}{\ell_1}\right)^2, \left(\frac{\partial \mathcal{E}_c^2}{\partial d}\right)^2 = \left(\frac{1}{\ell_1}\right)^2 \left(\frac{d_H - d}{2\ell_1}\right)^2 \\
&\left(\frac{\partial C}{C\partial d}\right)^2 = \left(\frac{4}{d^2}\right), \left(\frac{\partial \zeta}{\zeta\partial d}\right)^2 = 0\,.
\end{aligned}\right\} \quad (173)$$

Then:

$$\left.\begin{aligned}
\Delta_\beta X &= \left[+2900\left(\frac{\partial C}{C\partial \beta}\right)\right]\beta \\
\Delta_\beta Z &= \left[+101.8\left(\frac{\partial \zeta}{\zeta\partial \beta}\right)\right]\beta \\
\Delta_\beta \sigma_X^2 &= \left[8.410 \cdot 10^6 \left(\frac{\partial C}{C\partial \beta}\right)^2\right]\Delta\sigma_\beta^2 \\
\Delta_\beta \sigma_Z^2 &= \left[10363\left(\frac{\partial \zeta}{\zeta\partial \beta}\right)^2\right]\Delta\sigma_\beta^2\,.
\end{aligned}\right\} \quad (174)$$

$$\Delta_r X = \left[-5223\left(\frac{2\pi}{nd_H}\right) - 2.063\cdot10^6\left(\frac{2\pi}{nd_H}\right)^2\left(\left[\frac{d_H-d}{l_l}\right]l_g + r\right) + 2900\left(\frac{\partial C}{C\partial r}\right)\right] r$$

$$\Delta_r Z = \left[-9665\left(\frac{2\pi}{nd_H}\right) + 101.8\left(\frac{\partial\zeta}{\zeta\partial r}\right)\right] r \qquad 17$$

$$\Delta_r \sigma^2_X = \left[9.474\cdot10^7\left(\frac{2\pi}{nd_H}\right)^2 + 4.256\cdot10^{12}\left(\frac{2\pi}{nd_H}\right)^4\left(\left[\frac{d_H-d}{l_l}\right]l_g + r\right)^2 + 8.410\cdot10^6\left(\frac{\partial C}{C\partial r}\right)^2\right]\Delta\sigma^2_r$$

$$\Delta_r \sigma^2_Z = \left[5.457\cdot10^7\left(\frac{2\pi}{nd_H}\right)^2 + 10363\left(\frac{\partial\zeta}{\zeta\partial r}\right)^2\right]\Delta\sigma^2_r\,.$$

$$\Delta_b X = \left[5223\left(\frac{2\pi}{nd_H}\right)\left(\frac{d_H-d}{2\,l_l^2}\right)\left(l_l - l_g\right) - 4.56\left(\frac{2\pi}{nd_H}\right) - 2.063\cdot10^6\left(\frac{2}{l_l}\right)\left(\frac{d_H-d}{2\,l_l}\right)^2 + 2900\left(\frac{\partial C}{C\partial b}\right)\right]\varepsilon_b$$

$$\Delta_b Z = \left[9665\left(\frac{2\pi}{nd_H}\right)\left(\frac{d_H-d}{2\,l_l^2}\right)\left(l_l - l_g\right) + 1.42\left(\frac{2\pi}{nd_H}\right) + 101.8\left(\frac{\partial\zeta}{\zeta\partial b}\right)\right]\varepsilon_b \qquad 176$$

$$\Delta_b \sigma^2_X = \left[-9.474\cdot10^7\left(\frac{2\pi}{nd_H}\right)^2\left(\frac{d_H-d}{2\,l_l^2}\right)^2\left(l_l - l_g\right)^2 + 4.256\cdot10^{12}\left(\frac{2}{l_l}\right)^2\left(\frac{d_H-d}{2\,l_l}\right)^4 + 8.410\cdot10^6\left(\frac{\partial C}{C\partial b}\right)^2\right]\Delta\sigma^2_b$$

$$\Delta_b \sigma^2_Z = \left[-5.457\cdot10^7\left(\frac{2\pi}{nd_H}\right)^2\left(\frac{d_H-d}{2\,l_l}\right)^2\left(l_l - l_g\right)^2 + 10363\left(\frac{\partial\zeta}{\zeta\partial b}\right)^2\right]\Delta\sigma^2_b$$

$$\Delta_d X = \left[5223\left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right) + 2.063\cdot10^6\left(\frac{1}{l_l}\right)\left(\frac{d_H-d}{2\,l_l}\right) - 2900\left(\frac{2}{d}\right)\right]\varepsilon_d$$

$$\Delta_d Z = \left[9665\left(\frac{\pi}{nd_H}\right)\left(\frac{l_g}{l_l}\right)\right]\varepsilon_d$$

$$\Delta_d \sigma^2_X = \left[9.474\cdot10^7\left(\frac{\pi}{nd_H}\right)^2\left(\frac{l_g}{l_l}\right)^2 + 4.256\cdot10^{12}\left(\frac{1}{l_l}\right)^2\left(\frac{d_H-d}{l_l}\right)^2 + 8.410\cdot10^6\left(\frac{4}{d^2}\right)\right]\Delta\sigma^2_d \qquad 177$$

$$\Delta_d \sigma^2_Z = \left[5.457\cdot10^7\left(\frac{\pi}{nd_H}\right)^2\left(\frac{l_g}{l_l}\right)^2\right]\Delta\sigma^2_d\,.$$

Treatment of every dimension as normal except ε_L in the formulae (174), (175)...(177) yields the following theoretical effects:

$$\Delta_\beta X = \left[+2900\ \frac{\partial C}{C\partial\beta}\right]\beta$$

$$\Delta_\beta Z = \left[(101.8)\ \frac{\partial\zeta}{\zeta\partial\beta}\right]\beta^2$$

$$
\left.
\begin{aligned}
\Delta_{\varepsilon_\beta} \sigma_X^2 &= \left[\left(8.410 \cdot 10^6\right) \left(\frac{\partial C}{C \partial \beta}\right)^2 \right] \Delta\sigma_\beta^2 \\
\Delta_{\varepsilon_\beta} \sigma_Z^2 &= \left[\left(10363.\right) \left(\frac{\partial \zeta}{\zeta \partial \beta}\right)^2 \right] \Delta\sigma_\beta^2 \;.
\end{aligned}
\right\} \tag{178}
$$

$$
\left.
\begin{aligned}
\Delta_r X &= \left[\left(-269.8\right) + 2900 \left(\frac{\partial C}{C \partial r}\right) \right] r \\
\Delta_r Z &= \left[\left(-389.0\right) + \left(101.8\right) \left(\frac{\partial \zeta}{\zeta \partial r}\right) \right] r \\
\Delta_r \sigma_X^2 &= \left[\left(1.570 \cdot 10^5\right) + \left(8.410 \cdot 10^6\right) \left(\frac{\partial C}{C \partial r}\right)^2 \right] \Delta\sigma_r^2 \\
\Delta_r \sigma_Z^2 &= \left[\left(8.84 \cdot 10^4\right) + \left(10363.\right) \left(\frac{\partial \zeta}{\zeta \partial r}\right)^2 \right] \Delta\sigma_r^2 \;.
\end{aligned}
\right\} \tag{179}
$$

$$
\left.
\begin{aligned}
\Delta_b X &= \left[\left(-1.15\right) + 2900 \left(\frac{\partial C}{C \partial b}\right) \right] \varepsilon_b \\
\Delta_b Z &= \left[\left(0.2\text{[illegible]}\right) + \left(101.8\right) \left(\frac{\partial \zeta}{\zeta \partial b}\right) \right] \varepsilon_b \\
\Delta_b \sigma_X^2 &= \left[\left(1.10\right) + \left(8.410 \cdot 10^6\right) \left(\frac{\partial C}{C \partial b}\right)^2 \right] \Delta\sigma_b^2 \\
\Delta_b \sigma_Z^2 &= \left[\left(0.0120\right) + \left(10363.\right) \left(\frac{\partial \zeta}{\zeta \partial b}\right)^2 \right] \Delta\sigma_b^2 \;.
\end{aligned}
\right\} \tag{180}
$$

$$
\left.
\begin{aligned}
\Delta_d X &= \left[-515\right] \varepsilon_d \\
\Delta_d Z &= \left[145\right] \varepsilon_d \\
\Delta_d \sigma^2_X &= \left[1.06 \cdot 10^6\right] \Delta\sigma^2_d \\
\Delta_d \sigma_Z^2 &= \left[12347\right] \Delta\sigma^2_d \;.
\end{aligned}
\right\} \tag{181}
$$

The theoretical effects for the present shell are accordingly given by the following tables:

$\Delta_L X$ [YDS.]

	$\Delta_\beta X$	$\Delta_r X$	$\Delta_b X$	$\Delta_d X$
Standard	-	-	$-0.0599+150.8\frac{\partial C}{C\partial b}$	-0.6695
Savanna	$+91.35\frac{\partial C}{C\partial\beta}$	-	$+0.0081-20.3\frac{\partial C}{C\partial b}$	+0.1030
Charleston	-	$-17.42+187.3\frac{\partial C}{C\partial r}$	$-0.0058+14.5\frac{\partial C}{C\partial b}$	+2.214

	$\Delta_\beta Z$	$\Delta_r Z$	$\Delta_b Z$	$\Delta_d Z$
Standard	-	-	$+0.0105+5.294\frac{\partial\zeta}{\zeta\partial b}$	0.1885
Savanna	$+3.207\frac{\partial\zeta}{\zeta\partial\beta}$	-	$-0.0014-0.7126\frac{\partial\zeta}{\zeta\partial b}$	-0.0290
Charleston	-	$-25.12+6.574\frac{\partial\zeta}{\zeta\partial r}$	$+0.0010+0.5090\frac{\partial\zeta}{\zeta\partial b}$	-0.6235

	$\Delta_\beta\sigma_X^2$	$\Delta_r\sigma_X^2$	$\Delta_b\sigma_X^2$	$\Delta_d\sigma_X^2$
Standard	-	-	$+0.000,147+1126.\left(\frac{\partial C}{C\partial b}\right)^2$	-8.8
Savanna	$+891.\left(\frac{\partial C}{C\partial\beta}\right)^2$	-	$-0.000,022-168.2\left(\frac{\partial C}{C\partial b}\right)^2$	+14.9
Charleston	-	$+83.699+4483\left(\frac{\partial C}{C\partial r}\right)^2$	$+0.000,484+3700\left(\frac{\partial C}{C\partial b}\right)^2$	+53.5

	$\Delta_\beta\sigma_Z^2$	$\Delta_r\sigma_Z^2$	$\Delta_b\sigma_Z^2$	$\Delta_d\sigma_Z^2$
Standard	-	-	$+0.000,001,6+1.389\left(\frac{\partial\zeta}{\zeta\partial b}\right)^2$	-0.1025
Savanna	$+1.098\left(\frac{\partial\zeta}{\zeta\partial\beta}\right)^2$	-	$-0.000,000,2-0.2073\left(\frac{\partial\zeta}{\zeta\partial b}\right)^2$	+0.1741
Charleston	-	$+47.12+5.523\left(\frac{\partial\zeta}{\zeta\partial r}\right)^2$	$+0.000,00053+4.560\left(\frac{\partial\zeta}{\zeta\partial b}\right)^2$	+0.6235

IV. Data from Range Firing of Shell and Reduction of Observations

Under standard conditions the elements of the normal trajectory for normal shell ($C = 2.42$, $E = 23°15'$, $v_o = 1478$ [FT.SEC.$^{-1}$]) are:

$$\left.\begin{aligned} \bar{X} &= 9599\,[\text{YDS.}] \\ \bar{Z} &= 101.8\,[\text{YDS.}] \\ r_X &= 46\,[\text{YDS.}] \\ r_Z &= 6\,[\text{YDS.}]\,. \end{aligned}\right\} \qquad (182)$$

The mean maximum ordinate for the normal shell is:

$$Y_m = 1252\,[\text{YDS.}]\,. \qquad (183)$$

The range firing of the three samples of shell were carried out under the direction of Major C. F. Hofstetter on May 3, 1938. The results were recorded on Firing Record No. 11004 which is given in Appendix B of the present report.[1] The Standard, Savanna and Charleston rounds were alternated. Individual muzzle velocities, masses and times of firing were recorded for each shell. Individual observed ranges and deflections were measured. A plot of the observed ranges and deflections is given as Appendix C of the present report.

Complete meteorological data were taken at five different times during the day for wind and twice for temperature. The resultant measurements are given in Appendix D of the present report.

Determination of ballistic means for a maximum ordinate of 1250 yards in the usual fashion results in the following values:

t E.S.T.	$\bar{W}_x$ [MI. HR^{-1}]	$\bar{W}_z$ [MI. HR.$^{-1}$]	$\overline{\Delta H / H}$ [1]	$\bar{\tau}$ [°F]
$19^h.63$	+4.72	+1.12		
$10^h.70$	+2.83	+2.52		
$11^h.30$	[illegible]	[illegible]	0.9927	+7°.9
$12^h.07$	+0.01	+0.44		
$13^h.25$	-7.59	+3.42		
$14^h.13$	-4.46	+0.94		
$14^h.30$			0.9796	+9°.8

(184)

1 The correct mass reduction is that given in the body of this report and not the mass reduction given on Firing Record 11004.

These values are plotted as functions of the time and smooth curves drawn through them in Fig. 18, 19, 20, and 21. The heavy curves are the results of reduction of the Meteorological Data. The Ballistic Range and Cross Winds varied to a considerable degree during the firing. It will develop that the question raised in the 7th Indorsement of the quoted correspondence has this phenomenon at its root. That is, the fact "that the earliest rounds fired were generally longer then the mean, and later rounds fell short" is due largely to the presence of a rear wind at the beginning which became a head wind later in the day. The correspondence continues: "This would indicate that the projectiles having maximum variation from standard gave the greatest range. It is noted, however that this same condition existed for the so-called 'standard' projectiles - Lot 7884-1. It would therefore appear that some change occurred during the firing which progressively affected the range. The greatest degree of change appears to have occurred around noon. It is therefore requested that change in atmospheric conditions be investigated." It appears at once on comparison of the atmospheric changes which occurred during the noon hour that the atmospheric change referred to in the correspondence is that in the ballistic range wind. This wind had a strong minimum at this time.

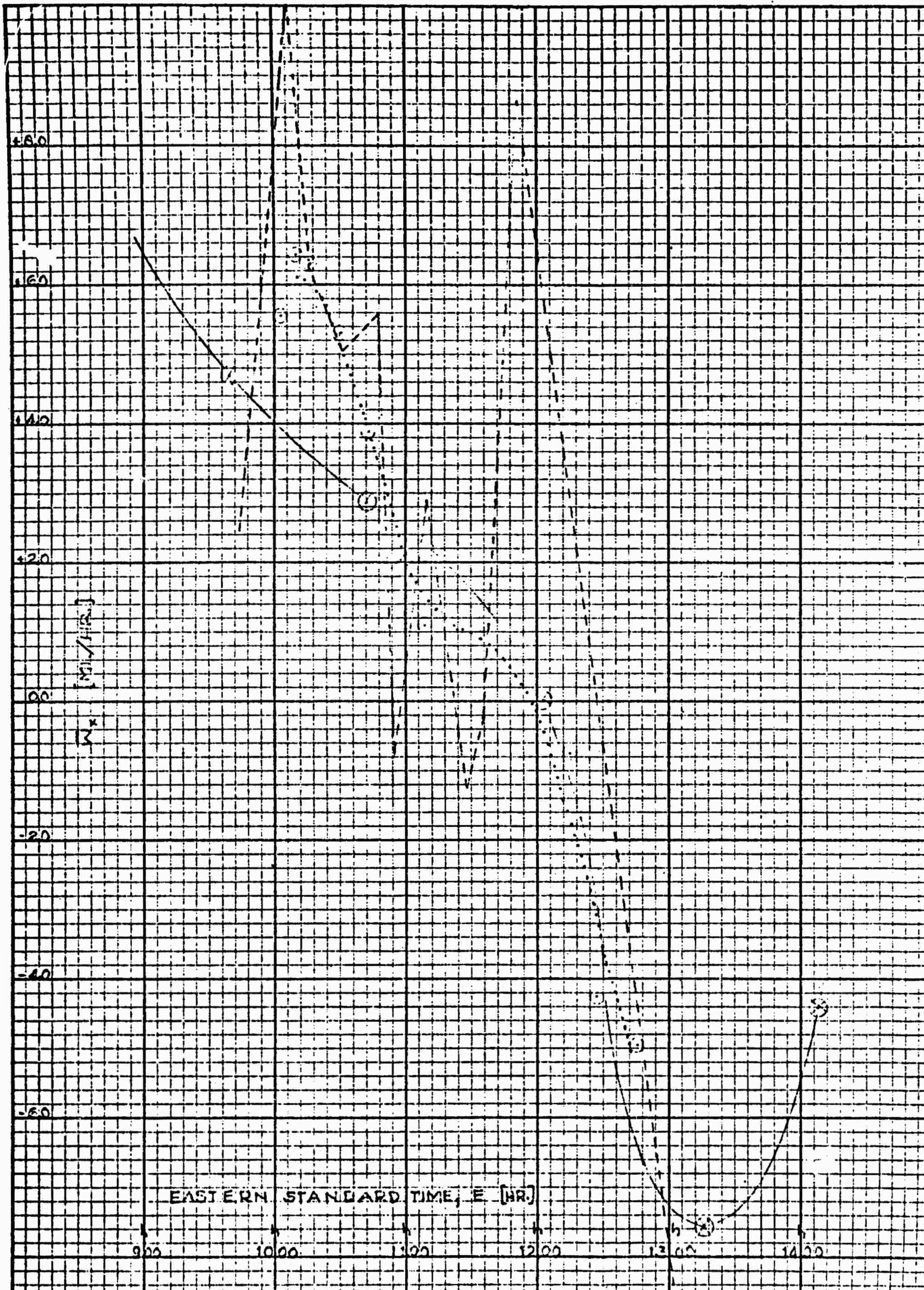
The effects of abnormalities are sought and these are in any case small. It was therefore considered desirable to carry out the reduction of observations for non-standard ballistic table conditions with utmost care. The reduction will be described in the following paragraphs, but for the present it will be advantageous to consider a verification of the shape and ordinates of the highly curved ballistic range and cross winds shown on the plots. This has been done in the following manner: the reduction of observations was carried out for all effects both including and excluding the ballistic range wind and cross wind. Let the ranges and deflections reduced for all non-standard conditions be denoted by X_β and Z_β, and let the ranges and deflections reduced for all non-standard conditions except $\bar{w}_x$ and $\bar{w}_z$ be denoted by $X_{\bar{w}_x}$ and $Z_{\bar{w}_z}$.

Let the normal rounds alone be considered: it has been supposed that these possess the population mean elements $\bar{X}$ and $\bar{Z}$[1] under standard ballistic table conditions. For the normal rounds it may be supposed that the quantities $\left(X_{\bar{w}_x} - \bar{X}\right)$ and $\left(Z_{\bar{w}_z} - \bar{Z}\right)$ contain only the effects of ballistic winds as functions of the time and accidental variations. Then, except for accidental errors:

[1] From equations (182) it is found that:

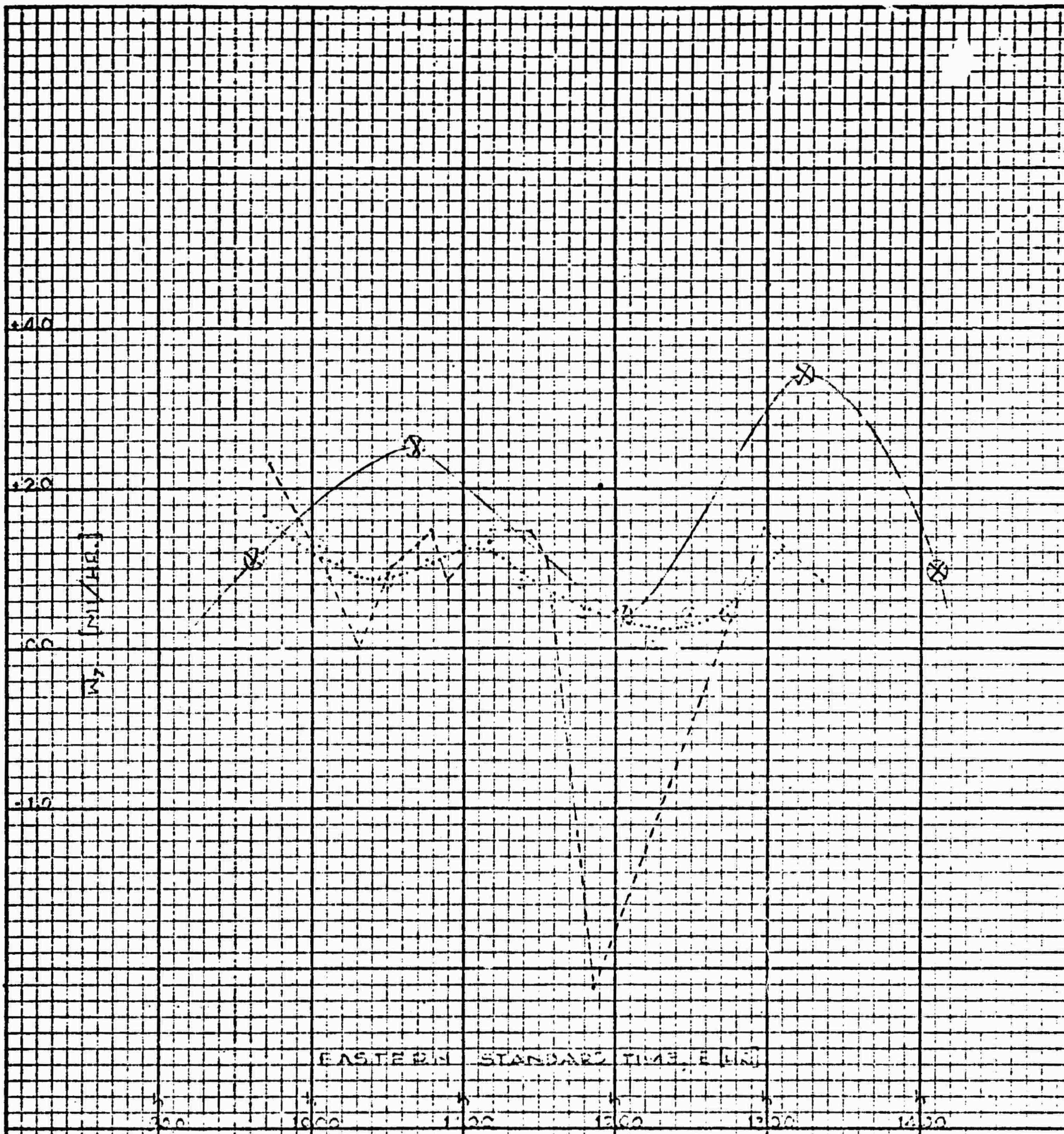
$$\bar{X} = 9599 \,[\text{YDS.}]$$

$$\bar{Z} = 101.8\,[\text{YDS.}]$$



$\overline{W}_x$: E

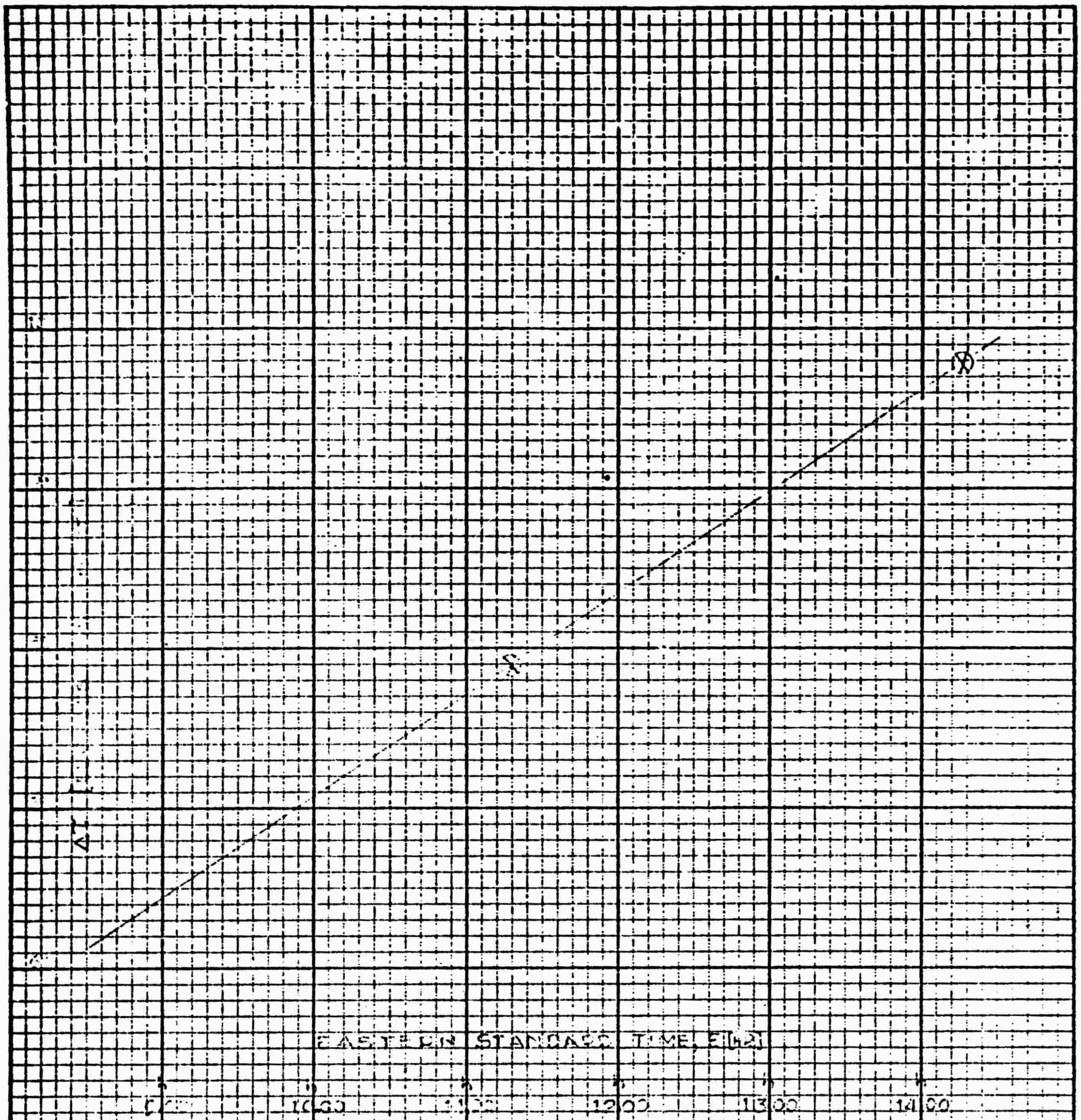
Fig 17



[illegible]

W_z : E

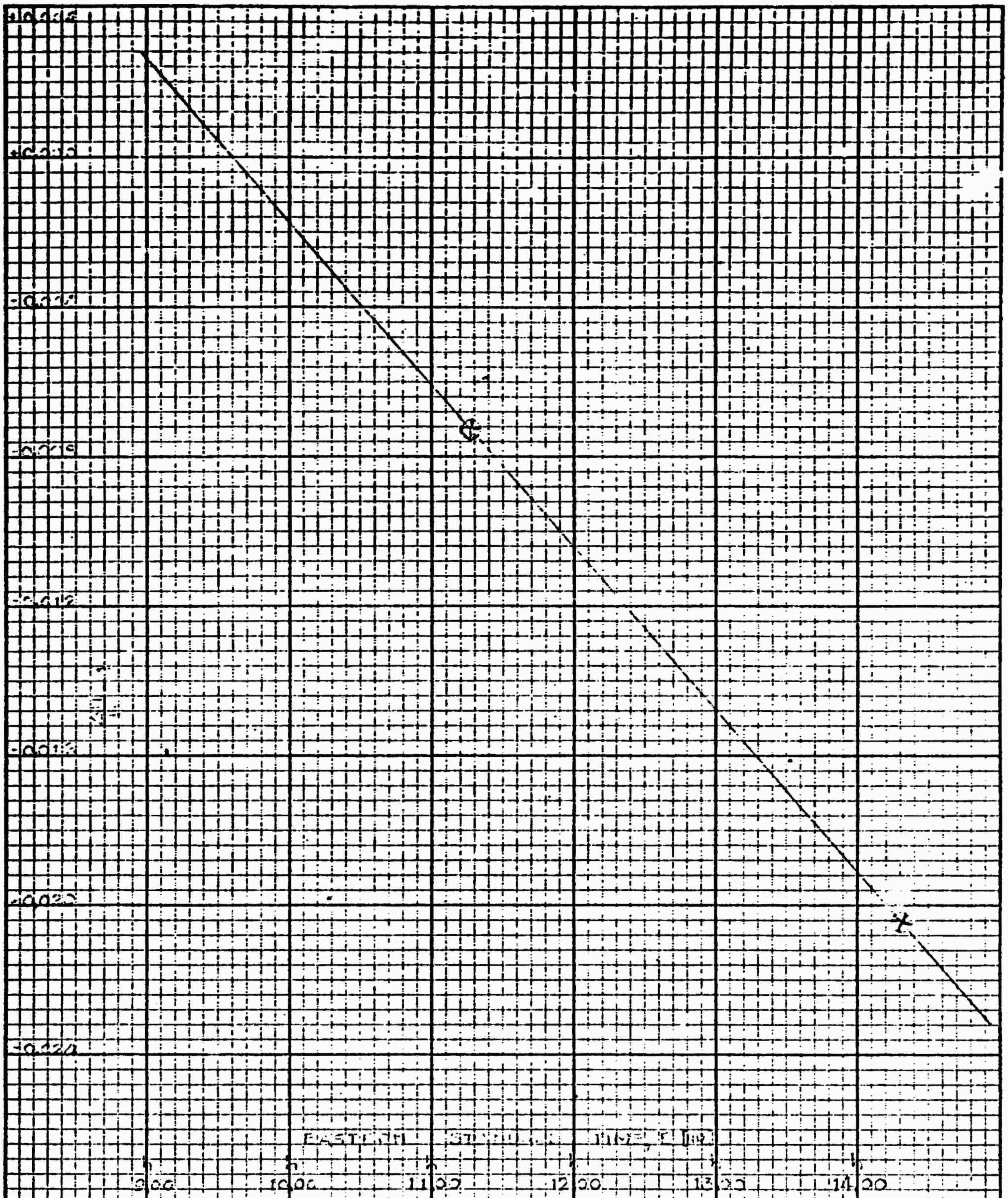
Fig. 19



DEPARTURE OF BALLISTIC TEMPERATURE FROM STANDARD AS A FUNCTION OF TIME OF FIRING

ΔT:E

Fig. 20



DEPARTURE OF [illegible] DENSITY FROM STANDARD
[illegible] TIME OF FIRING

Fig. 21

$$\left.\begin{aligned} \bar{w}_x &= \frac{X_{\bar{w}_x} - \bar{X}}{11.3} \left[\text{MI. HR}^{-1} \right] \\ \bar{w}_z &= \frac{Z_{\bar{w}_z} - \bar{Z}}{4.8} \left[\text{MI. HR}^{-1} \right]. \end{aligned}\right\} \qquad (185)$$

Theseresults are plotted as broken straight lines in the Figs. 18 and 19. In order to remove the accidental errors the results were smoothed numerically and are shown as the dotted curves on these diagrams.

Comparison of the ballistic range and cross winds computed from the meteorological data and shown as full lines and the ballistic range and cross winds deduced from the results of reduction of observations for all effects but wind shows a remarkable agreement. The wind effects might have been determined from the latter procedure as well as the former. Then the small effects sought herein will not be spurious due to the presence of badly determined wind effects.[1]

The non-standard ballistic table conditions present in the case of these shell are departure from standard mass, departure from standard muzzle velocity, departure from standard temperature of powder, departure from zero ballistic range wind, departure from zero ballistic cross wind, departure from normal ballistic density, departure from normal ballistic temperature, cant, departure from zero depth of plane of impact below the trunnions and rotation of the earth. In order to afford comparison with firing table values for certain purposes all of the effects of these non-standard conditions except those due to rotation of the earth have been removed in the process of reduction of observations on every round of the three samples. The quantities removed are:

Range Effects

$\Delta_{mc} X$: Mass effect on range through mass effect on ballistic coefficient;

$\Delta_{mv_o} X$: Mass effect on range through mass effect on muzzle velocity;

$\Delta_{v_o} X$: Effect on range due to non-zone muzzle velocity;

$\Delta_p X$: Effect of temperature of powder on range;

$\Delta_{W_x} X$: Effect of ballistic range wind on range;

$\Delta_H X$: Effect of ballistic density on range;

$\Delta_\tau X$: Effect of ballistic temperature on range;

$\Delta_h X$: Effect of depth of point of impact on range.

[1] The method of verifying the wind effects directly from the results of observations was suggested to the writer by Mr. R.H. Kent.

Deflection Effects

$\Delta_{W_z} Z'$: Effect of ballistic cross wind on deflection;

$\Delta_k Z$: Effect of cant on deflection.

Let X_n denote the unreduced range, and X_β the range reduced to standard ballistic table conditions. A similar notation will be used for deflection. Evidently:

$$\left.\begin{aligned} X_\beta &= X_n - \sum_n \Delta_n X \\ Z_\beta &= Z_n - \sum_n \Delta_n Z . \end{aligned}\right\} \qquad (186)$$

It will be noted that the removal of the mass effects is in any case obviously necessary since the standard projectiles were heavier than the corresponding eccentric projectiles. In addition the heavier rounds were in general fired earlier than the lighter. It is necessary to reduce the range for change in mass both as respects the effect of mass on ballistic coefficient and on muzzle velocity. From the firing table[1] it is found that the mass of projectile and M46 Fuze is 94.7 [LBS.], and that the change in range for one per cent change in ballistic density is -29 yds. Then:

$$\left.\begin{aligned} \Delta m &= m - m_s = (m - 94.7)\,[\text{LBS.}] \\ \Delta_{m_c} X &= +2900 \frac{\Delta m}{m_s}\,[\text{YDS.}]. \end{aligned}\right\} \qquad (187)$$

The effect of mass on muzzle velocity is obtainable from the equation:

$$\delta v_o = \pm\, n \frac{\Delta m}{m_s} v_o{}^{2}\,[\text{FT.SEC.}^{-1}]. \qquad (188)$$

The value of the quantity n for these shell at the muzzle velocity 1478 [FT.SEC.$^{-1}$] is 0.305.[3] The muzzle velocity corrected for mass is denoted by $v_{c_{\Delta v_o}}$ and computed by:

$$v_{o_{\Delta v_o}} = v_{o_n} - \delta v_o\,[\text{FT. SEC.}^{-1}].$$

The range effect of the effect on muzzle velocity due to the augumentation of mass is:

$$\Delta_{m_{v_o}} X = +6.6\,\delta v_o\,[\text{YDS.}]. \qquad (189)$$

[1] 155-D-3
[2] H.P. Hitchcock: Computation of Firing Tables for the U.S. Army.
[3] This value was given to the writer by R.H. Kent .

The actual velocities having been corrected for mass and the range effect corresponding having been found, it is necessary to consider the departure of the reduced velocity, $v_{o\Delta v_o}$ from standard, $v_o = 1478\,[\text{FT.SEC.}^{-1}]$ Then:

$$\Delta v_o = v_{o\Delta v_o} - 1478\,[\text{FT.SEC.}^{-1}]. \qquad (190)$$

From the firing table it is found that, for this zone and elevation:

$$\Delta v_o X = +6.6\,\Delta v_o\,[\text{YDS.}]. \qquad (191)$$

The standard temperature of powder for these shell is found to be 70°F and the observed temperature 71°F. It appears from the firing table that the muzzle velocity effect of 1°F. departure from standard ballistic table conditions is negligible.

The use of the firing table effects results in the following meteorological effects:

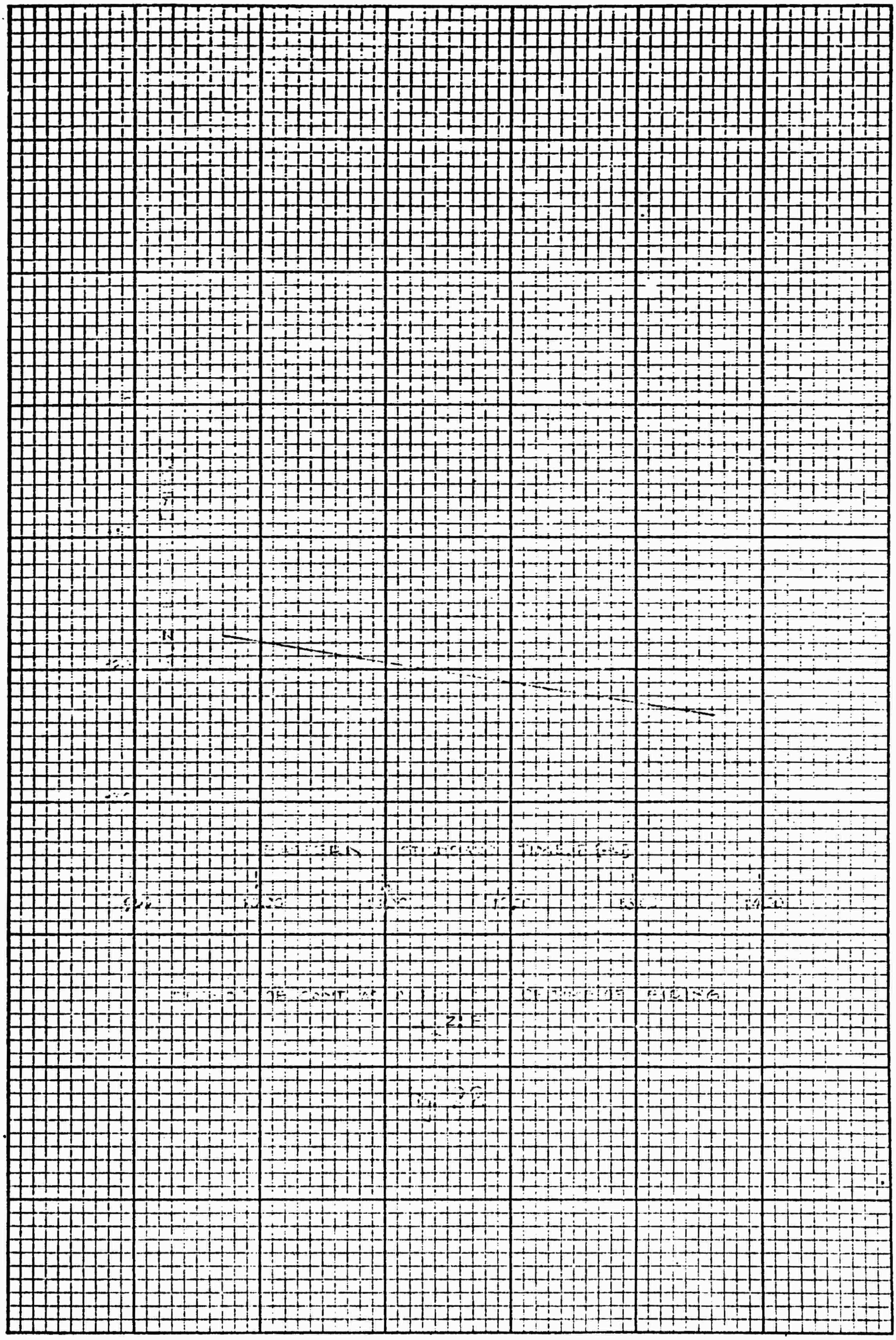
$$\left.\begin{aligned} \Delta_{\bar{w}_x} X &= +11.3\bar{w}_x\,[\text{YDS.}] \\ \Delta_H X &= -2900\left(\frac{\overline{\Delta H}}{H}-1\right)[\text{YDS.}] \\ \Delta_\tau X &= +5.0\,\overline{\Delta\tau}\,[\text{YDS.}]. \end{aligned}\right\} \qquad (192)$$

The points of impact on the range are on a nearly level field 12.7 feet above mean low water. The height of the trunnions of the howitzer above mean low water was equal to 38.5 feet. The difference in elevation, h, howitzer above impact, is therefore 8.6 yards. From the firing table it is found that the slope of fall is 1/1.72 for a 9600 yard range and 1/1.75 for a 9500 yard range. These quantities result in effects of 14.8 yards and 15.1 yards. The difference is negligible, whence

$$\Delta_h X = h \cot \omega = +15\,[\text{YDS.}]. \qquad (193)$$

The firing record 11004 furnishes the following information:

"Cant of right axle before Rd. 2133. Right trunnion above left = 0.009 ft. After firing, right trunnion above left 0.012 ft. Distance between points 2.14 ft." Then before firing, the cant, k, was 4.2 $\not{m}$ and after firing, 5.6 $\not{m}$. From the firing table it is found that the deflection effect of 10 mils cant is 4.4 $\not{m}$ for 9600 yards and 4.3 $\not{m}$ for 9500 yards. Accordingly the mean effect of the cant before the beginning of the firing was 17.4 yards and after the conclusion of firing 23.2 yards. Exact knowledge of the augmentation of cant during firing islacking.
lacking.



It appears desirable to assume that this augmentation proceeds uniformly with the time. Fig. 22 shows the effect, $\Delta_K Z$ as if linearly related to the time of firing.

The cross-wind effect on deflection for a 1 [MI.HR.$^{-1}$] wind at these ranges is 0.5 mil.

Then for the normal shell:

$$\Delta_{\bar{\bar{w}}_z} Z = 4.8\ \bar{\bar{w}}_Z \quad [\text{YDS.}] \tag{194}$$

For the eccentric shell:

$$\Delta_{\bar{\bar{w}}_z} Z = 4.75 \bar{\bar{w}}_z \quad [\text{YDS.}]. \tag{195}$$

The following table gives these results in compact form. It will be noted that there is greater uniformity between the results for the ranges and deflections reduced to standard conditions between the three groups than among the corresponding unreduced ranges and deflections.

Round No.	E Eastern Standard Time of Firing	M Mass Including Fuze	X_n	$\Delta m_c X$	$\Delta mv_o X$	$\Delta v_o X$	$\Delta w_x X$	$\Delta_H X$	$\Delta_\tau X$	$\Delta_h X$	X_β	Z_n	$\Delta w_z Z$	$\Delta_k Z$	Z_β
		[LBS.]	[YDS.]	[YDS.]	[YDS.]	[YDS.]	[YDS.]	[YDS.]	[YDS.]		[YDS.]	[YDS.]	[YDS.]	[YDS.]	[YDS.]
Standard 2133	$9^h.73$	96.6	9683	+58	-59	+7	+51	+1	+34	15	9576	96	+7	-17	106
2136	$10^h.12$	96.6	9715	+58	-59	-53	+43	+6	+36	15	9669	88	+10	-18	96
2139	$10^h.32$	96.6	9682	+58	-59	-46	+39	+9	+36	15	9630	84	+11	-18	91
2142	$10^h.55$	96.6	9706	+58	-59	-13	+35	+12	+37	15	9621	88	+12	-19	95
2145	$10^h.80$	95.0	9646	+9	-7	-86	+31	+15	+38	15	9631	90	+12	-19	97
2148	$10^h.92$	96.5	9636	+55	-53	-26	+30	+16	+38	15	9561	87	+11	-19	95
2151	$11^h.17$	96.5	9608	+55	-53	-99	+26	+19	+39	15	9606	89	+8	-20	101
2154	$11^h.45$	96.5	9599	+55	-53	-66	+20	+23	+40	15	9565	89	+6	-20	103
2157	$11^h.57$	96.5	9625	+55	-53	-53	+17	+24	+40	15	9580	87	+5	-20	102
2160	$11^h.87$	96.5	9630	+55	-53	-152	+7	+28	+41	15	9689	70	+3	-21	88
2165	$12^h.98$	96.5	9615	+55	-53	+7	-82	+42	+45	15	9586	87	+14	-22	95
2170	$13^h.38$	96.5	9487	+55	-53	-59	-85	+47	+46	15	9521	83	+16	-23	90
Arith. Mean	$11^h.24$	96.4	9636	+52.2	-51.2	-53.2	+11.0	+20.2	+39.2	15	9602.9	86.5	+9.6	-19.7	96

Round No.	E Eastern Standard Time of Firing	M Mass Including Fuze [LBS.]	X_n [YDS.]	$\Delta m_c X$ [YDS.]	$\Delta mv_o X$ [YDS.]	$\Delta v_o X$ [YDS.]	$\Delta w_x X$ [YDS.]	$\Delta_H X$ [YDS.]	$\Delta_\tau X$ [YDS.]	$\Delta_h X$	X_β [YDS.]	Z_n [YDS.]	$\Delta w_z Z$ [YDS.]	$\Delta_k Z$ [YDS.]	Z_β [YDS.]
Savanna															
2134	$9^h.85$	95.4	9573	+20	-20	-92	+48	+3	+34	15	9565	78	+8	-18	88
2137	$10^h.22$	95.3	9663	+17	-20	-40	+40	+8	+35	15	9608	79	+10	-18	87
2140	$10^h.48$	95.0	9535	+9	-7	-158	+36	+11	+36	15	9593	86	+12	-19	93
2143	$10^h.58$	95.1	9579	+12	-13	-119	+34	+12	+36	15	9602	86	+12	-19	93
2146	$10^h.85$	94.5	9524	-6	+7	-86	+30	+15	+37	15	9512	80	+11	-19	88
2149	$11^h.08$	93.8	9472	-28	+26	-165	+27	+18	+38	15	9541	85	+9	-20	96
2152	$11^h.20$	94.3	9542	-12	+13	-79	+25	+20	+38	15	9522	76	+8	-20	88
2155	$11^h.50$	94.5	9520	-6	+7	-152	+18	+23	+39	15	9576	77	+6	-20	91
2158	$11^h.62$	93.8	9540	-28	+26	-53	+16	+25	+40	15	9499	80	+5	-20	95
2161	$11^h.90$	94.8	9520	+3	0	-152	+6	+29	+41	15	9578	66	+3	-21	84
2163	$12^h.02$	93.2	9423	-46	+46	-106	+2	+30	+41	15	9441	82	+2	-21	101
2166	$13^h.03$	94.0	9415	-20	+20	-99	-82	+43	+44	15	9494	68	+15	-23	76
2168	$13^h.12$	94.3	9497	-12	+13	-66	-84	+44	+44	15	9543	55	+16	-23	62
2171	$13^h.42$	94.4	9501	-9	+7	-40	-83	+48	+45	15	9518	69	+16	-23	76
2173	$13^h.48$	94.3	9423	-12	+13	-178	-82	+48	+46	15	9573	66	+15	-23	74
Arith. Mean	$11^h.62$	94.4	9515.1	-7.9	+7.9	-105.7	-3.3	+25.1	+39.6	15	9544.3	75.5	+9.9	-20.5	86.

Round No.	E Eastern Standard Time of Firing	M Mass Including Fuze [LBS.]	X_n [YDS.]	$\Delta m_c X$ [YDS.]	$\Delta mv_o X$ [YDS.]	$\Delta v_o X$ [YDS.]	$\Delta w_x X$ [YDS.]	$\Delta_H X$ [YDS.]	$\Delta_T X$ [YDS.]	$\Delta_h X$	X_β [YDS.]	Z_n [YDS.]	$\Delta w_z Z$ [YDS.]	$\Delta_k Z$ [YDS.]	Z_β
Charleston															
2135	$9^h.92$	94.7	9556	0	0	-92	+46	+4	+34	15	9549	81	+8	-18	91
2138	$10^h.28$	95.1	9459	+12	-13	-46	+39	+8	+36	15	9408	105	+11	-18	112
2141	$10^h.52$	94.0	9518	-20	+20	-73	+35	+11	+36	15	9494	97	+12	-19	104
2144	$10^h.75$	93.8	9604	-28	+26	-112	+32	+14	+37	15	9620	81	+12	-19	88
2147	$10^h.88$	95.7	9468	+32	+33	-33	+29	+16	+37	15	9405	88	+11	-19	96
2150	$11^h.13$	93.6	9433	-35	+33	-178	+26	+19	+38	15	9515	91	+9	-20	102
2153	$11^h.23$	93.1	9367	-49	+53	-198	+24	+20	+38	15	9464	82	+8	-20	94
2156	$11^h.53$	93.2	9379	-46	+46	-125	+18	+24	+39	15	9408	82	+6	-20	96
2159	$11^h.83$	93.6	9365	-35	+33	-165	+9	+28	+40	15	9440	87	+3	-21	105
2162	$11^h.95$	94.1	9497	-17	+20	-59	+4	+29	+41	15	9464	83	+2	-21	102
2164	$12^h.90$	93.2	9380	-46	+46	-86	-78	+41	+44	15	9444	63	+13	-22	72
2167	$13^h.08$	93.8	8779	-28	+26	-158	-83	+44	+44	15	8919	77	+15		
2169	$13^h.17$	93.7	9521	-32	+33	-66	-84	+45	+45	15	9565	58	+16	-23	65
2172	$13^h.45$	95.1	9513	+12	-13	-125	-82	+48	+45	15	9613	63	+15	-23	71
2174	$13^h.53$	94.2	9442	-15	+13	-92	-81	+49	+46	15	9507	66	+15	-23	74
Arith. Mean[1]	$11^h.74$	94.1	9418.7	-19.7	+19.3	-107.2	-9.7	+26.7	+40.0	15	9654.3	80.3	+10.4		
Arith. Mean[2]	$11^h.65$	94.1	9464.4	-19.1	+18.9	-103.6	-4.5	+25.4	+39.7	15	9492.6	80.5	+10.1	-20.4	90.9

[1] With 21675
[2] Not 2167

V. Tests of Significance of Range, Deflection and Dispersion Effects of Eccentricity of Shell

The foregoing mean points of impact and dispersion for the normal and the eccentric shell are:[1]

	$\bar{X}$	$\bar{Z}$	$\widehat{S_X^2}$	$\widehat{S_Z^2}$
Standard	9602.9	96.6	2089.8	27.91
Savanna	9544.3	86.1	2043.4	97.72
Charleston	9492.6	90.9	4901.1	[illegible].98

The significance of the apparent differences requires investigation first. The significance of the apparent system of reduced ranges and deflections for the three groups will be examined later. An analysis of variance with reference to order and to kind of abnormality will be employed first.

[1] Charleston round 2167, which had a range 700 yards short of the mean has been rejected in the calculation of all statistics from this point forward.

The shell are ranked according to nearly corresponding times, 12 shell in each group. With a constant subtracted from the ranges. and a true listing of deflections, the following values result.

GROUP RANK	ε_{STD}	ε_{SA}	ε_{CH}	$\Sigma\varepsilon$ υ	$\Sigma\varepsilon^2$ 1	[illegible] 2
1	291	280	264	835	232777	697225
2	384	323	123	830	266914	688900
3	345	308	209	862	257570	743044
4	336	317	335	988	325610	976144
5	346	227	120	693	185645	480249
6	276	256	230	762	194612	580644
7	321	237	179	737	191251	543169
8	280	291	123	694	178210	481636
9	295	214	155	664	156846	440896
10	404	293	179	876	281106	767376
11	301	156	159	616	140218	379456
12	236	209	280	725	177777	525625
$V = \Sigma\varepsilon$	3815	3111	2356	R= 9282	P=	$Y = \Sigma\upsilon^2$
$V' = \Sigma\varepsilon^2$	1237929	835659	514948		2588536	7304364
V^2	14554225	9678321	5550736	$C = \Sigma V^2 =$	29783282	

$R^2 = 86155524$ $\qquad (1/36)R^2 = 2393209.00$

$(1/12)C = 2481940.16$ $\qquad (1/3)\,Y = 2434788.00$

SOURCES	SUM OF SQUARES	DEGREES OF FREEDOM	VARIANCE	STANDARD DEVIATION	
BETWEEN SAMPLES	88731.16	2	44365.58	210.631	s_1
BETWEEN TIMES	41579.00	11	3779.909	61.481	s_2
RESIDUAL	65016.84	22	2955.3109	54.363	s_3
TOTAL	195327.00	35			

$Z_1 = 1.3544$ $\qquad Z_2 = 0.1231$

$0.001 < P(Z_1) < 0.01$ $\qquad P(Z_2) > 0.05$

GROUP RANK	Z_{STD}	Z_{SA}	Z_{CH}	ΣZ ν	ΣZ^2 ν^2	
1	106	88	91	285	27261	81225
2	96	87	112	295	29329	87025
3	91	93	104	288	27746	82944
4	95	93	88	276	25418	76176
5	97	88	96	281	26369	78961
6	95	96	102	293	28645	85849
7	101	88	94	283	26781	80089
8	103	91	96	290	28106	84100
9	102	95	105	302	30454	91204
10	88	84	102	274	25204	75076
11	95	101	72	268	24410	71824
12	90	76	65	231	18101	53361

$V = \Sigma Z$	1159	1080	1127	$R=$ 3366	$P=$	$\bar{Y} = \Sigma \nu^2$
$V^1 = \Sigma Z^2$	112275	97654	107895		317824	947834
V^2	1343281	1166400	1270129	$C = \Sigma V^2 = 3779810$		

R^2 11329956 $\qquad (1/36)R^2 = 314721.00$

$(1/12)C = 314984.166 \qquad (1/3)\bar{Y} = 315944.666$

SOURCES	SUM OF SQUARES	DEGREES OF FREEDOM	VARIANCE	STANDARD DEVIATION	
BETWEEN SAMPLES	263.166	2	131.583	11.471	s_1
BETWEEN TIMES	1223.666	11	111.242	10.547	s_2
RESIDUAL	1616.168	22	73.46218	8.5710	s_3
TOTAL	310330000	35			

$Z_1 = 0.2914 \qquad Z_2 = 0.2075$

$P(Z_1) > 0.05 \qquad P(Z_2) > 0.05$

It appears that the effects of eccentricity of mass, eccentricity of boattail and other departures from normality are significant as regards the groups of ranges but not as regards the times between the ranges. Then it appears that the group of deviations in range differ as to cause but are not associated significantly with the time of firing. With regard to the deflections it appears dubious whether the deviation groups are significantly different as regards either the sample or the time, although more influence appears to arise from the sample. Accordingly the samples are adjudged as having different population mean ranges. Possibly there are different population mean deflections, although the latter is uncertain.

Several other considerations regarding the significance of the differences may be alluded to briefly. By the Student "t" test:

$$\Delta\bar{X}_{ST-SA} = \bar{X}_{ST} - \bar{X}_{SA} = 58.7 \text{ [YDS.]} ; \quad t = 3.210; \ P_t < 0.01$$

$$\Delta\bar{X}_{ST-CH} = \bar{X}_{ST} - \bar{X}_{CH} = 110.4 \text{ [YDS.]} ; \quad t = 4.492; \ P_t < 0.01$$

$$\Delta\bar{X}_{SA-CH} = \bar{X}_{SA} - \bar{X}_{CH} = 51.7 \text{ [YDS.]} ; \quad t = 2.295; \ P_t \doteq 0.03$$

$$\Delta\bar{Z}_{ST-SA} = \bar{Z}_{ST} - \bar{Z}_{SA} = 10.5 \text{ [YDS.]} ; \quad t = 3.194; \ P_t < 0.01$$

$$\Delta\bar{Z}_{ST-CH} = \bar{Z}_{ST} - \bar{Z}_{CH} = 5.7 \text{ [YDS.]} ; \quad t = 1.259; \ 0.20 < P_t < 0.30$$

$$\Delta\bar{Z}_{SA-CH} = \bar{Z}_{SA} - \bar{Z}_{CH} = -4.8 \text{ [YDS.]} ; \quad t = -1.023; \ 0.30 < P_t < 0.40$$

It appears by these tests that the mean ranges of the samples are significantly different and that the mean deflections of the Standard and Savanna shell are probably different. The case for a significant difference in the mean deflections of Standard and Charleston, or Savanna and Charleston Shell is very weak.

The estimates of the population standard deviations are:

$$S_{X_{STD_1}} = 48.8 \text{ [YDS.]}$$

$$S_{X_{SA_1}} = 47.6 \text{ [YDS.]}$$

$$S_{X_{CH_1}} = 74.1 \text{ [YDS.]}$$

$$S_{Z_{STD_1}} = 5.6 \text{ [YDS.]}$$

$$S_{Z_{SA_1}} = 10.4 \text{ [YDS.]}$$

$$S_{Z_{CH_1}} = 15.1 \text{ [YDS.]}.$$

The differences and their standard errors yield

ΔS_1 (YDS)	$\sigma_{\Delta S_1}$	t	P_t [1]
$S_{X_{STD_1}} - S_{X_{SA_1}} = 1.2$	13.3	0.09	——
$S_{X_{STD_1}} - S_{X_{CH_1}} = -25.2$	17.2	-1.465	$<$ 0.21
$S_{X_{SA_1}} - S_{X_{CH_1}} = -26.4$	16.5	-1.600	$<$ 0.17
$S_{Z_{STD_1}} - S_{Z_{SA_1}} = -4.8$	2.2	-2.182	$<$ 0.09
$S_{Z_{STD_1}} - S_{Z_{CH_1}} = -9.4$	3.0	-3.133	$<$ 0.05
$S_{Z_{SA_1}} - S_{Z_{CH_1}} = -4.7$	3.4	-1.382	$<$ 0.23.

[1] The probability inequalities are based on the Camp-Meidel Inequality.

The consequences are that no significant differences between the standard deviations in range between the three samples are indicated. It is indicated that the standard deviations in deflection of Standard and Savanna Shell and of Standard and Charleston Shell are probably different. No significant difference is indicated between Charleston and Savanna Shell as regards the standard deviations in deflection. Somewhat similar results as regards both the significance of the differences of sample means and standard deviations appear if successive difference statistics are used to eliminate the possible influence of remaining unremoved fluctuations in the wind. It appears somewhat undesirable to base conclusions for the present shell on the successive difference statistics however, as the degree of asymmetry and abnormality of the different shell varies systematically from round to round in the case of the abnormal shell. This variation takes place largely in the direction of decreasing abnormality with time of firing.

B. <u>The Effects as Functions of the Eccentricity</u>

The fact that several abnormalities in dimension are present for each shell of the three samples results in a requirement for the separation of the effects. These abnormalities and the difference of actual and firing table ranges and deflections are given in the following table. The eccentricities of boattail and mass in the normal shell are unknown, because they were unmeasured. It is likely that these quantities were small in comparison to the corresponding eccentricities of abnormal shell and they are therefore listed as zero.

Round Number	ξ =X-9599 [YDS.]	ζ =(Z-101.8) [YDS.]	β [IN.]	(δm) [OZ.]	ε_b=(b-3.17) [IN.]	ε_d=(d-6.078) [IN.]
Standard						
2133	- 23	4.2	0.000	0	0.05	0.001
2136	70	- 5.8			0.06	0.002
2139	31	- 10.8			0.03	-0.001
2142	22	- 6.8			0.05	0.001
2145	32	- 4.8			0.05	0.001
2148	- 38	- 6.8			0.03	0.002
2151	7	- 0.8			0.07	0.002
2154	- 34	1.2			0.07	0.003
2157	- 19	0.2			0.03	0.000
2160	90	- 13.8			0.05	0.000
2165	- 13	- 6.8			0.06	0.001
2170	- 78	- 11.8	0.000	0	0.08	0.004
Savanna						
2134	- 34	- 13.8	0.045	0	0.00	-0.001
2137	9	- 14.8	0.040		0.00	0.002
2140	- 6	- 8.8	0.047		-0.02	0.003
2143	3	- 8.8	0.044		-0.01	0.003
2146	- 87	- 13.8	0.030		0.02	-0.001
2149	- 58	- 5.8	0.030		0.01	0.002
2152	- 77	- 13.8	0.043		-0.01	-0.002
2155	- 23	- 10.8	0.015		-0.02	0.000
2158	- 100	- 6.8	0.030		-0.01	0.004
2161	- 21	- 17.8	0.035		-0.01	0.004
2163	- 158	- 0.8	0.033		-0.01	-0.004
2166	- 105	- 25.8	0.020		-0.01	0.000
2168	- 56	- 39.8	0.023		-0.01	-0.016
2171	- 81	- 25.8	0.020		-0.01	0.001
2173	- 26	- 27.8	0.018	0	-0.02	0.002
Charleston						
2135	- 50	- 10.8	0.000	62	0.04	0.000
2138	- 191	10.2		58	-0.01	-0.005
2141	- 105	2.2		34	-0.02	0.002
2144	21	- 13.8		34	0.02	-0.003
2147	- 194	- 5.8		28	-0.02	-0.020
2150	- 84	0.2		28	0.02	0.003
2153	- 135	- 7.8		27	0.06	-0.021
2156	- 191	- 5.8		27	0.00	-0.002
2159	- 159	3.2		27	0.02	-0.012
2162	- 135	0.2		26	-0.01	0.001
2164	- 155	- 29.8		26	0.00	-0.002
2167	- 680			26	0.00	-0.008
2169	- 34	- 36.8		26	-0.01	-0.002
2172	14	- 30.8		26	0.00	0.001
2174	- 92	- 27.8	0.000	25	-0.02	0.000
Σ(Not inc. 2167)	-2263	-450.8	0.473	454	0.59	-0.047

The coefficients c_β, $c_{\delta m}$, c_b, k_β, $k_{\delta m}$, k_b and k_d are determined from the experimental data. By the reason of the property of linear homogeneity of the effects in the abnormalities, the coefficients satisfy the following equations:

$$\left.\begin{aligned}
\Delta_\beta X &= \frac{dX}{d\beta}\beta = c_\beta\beta \; ; \; \Delta_\beta Z = \frac{dZ}{d\beta}\beta = k_\beta\beta \\
\Delta_{\delta m} X &= \frac{dX}{d\delta m}\delta m = c_{\delta m}\,\delta m ; \; \Delta_{\delta m} Z = \frac{dZ}{d\delta m}\delta m = k_{\delta m}\delta m \\
\Delta_b X &= \frac{dX}{db}\varepsilon_b = c_b\,\varepsilon_b ; \; \Delta_b Z = \frac{dZ}{db}\varepsilon_b = k_b\,\varepsilon_b \\
\Delta_d X &= \frac{dX}{dd}\varepsilon_d = c_d\,\varepsilon_d ; \; \Delta_d Z = \frac{dZ}{dd}\varepsilon_d = k_d\,\varepsilon_d \,.
\end{aligned}\right\} \quad (196)$$

The normal equations providing for the determination of these coefficients by the method of least squares are derived from observational equations of the form:

$$\left.\begin{aligned}
v_\xi &= \xi_c - \xi_o = c_\beta\beta + c_{\delta m}\delta m + c_b\,\varepsilon_b + c_d\,\varepsilon_d - \xi_o \\
v_\zeta &= \zeta_c - \zeta_o = k_\beta\beta + k_{\delta m}\delta m + k_b\,\varepsilon_b + k_d\,\varepsilon_d - \zeta_o ,
\end{aligned}\right\} \quad (197)$$

The simultaneous minimization of $\sum v_\xi^2$ with respect to the unknowns c_β, $c_{\delta m}$, c_b and c_d yields the following system of normal equations:

$$0.016511\, c_\beta + 0.0\, c_{\delta m} - 0.00318\, c_b + 0.000068\, c_d = -23.751$$

$$0.0\, c_\beta + 16604\, c_{\delta m} + 3.04\, c_b - 1.797\, c_d = -48273$$

$$-0.00318\, c_\beta + 3.04\, c_{\delta m} + 0.0465\, c_b - 0.00005\, c_d = 2.17$$

$$0.000068\, c_\beta - 1797\, c_{\delta m} - 0.00005\, c_b + 0.001429\, c_d = 10.422$$

$$c_d = 4263.32 \pm 1139.54 \qquad (r_{c_d}) \; [\mathrm{YDS.\ IN.}^{-1}]$$

$$c_b = 114.46 \pm 188.03 \qquad (r_{c_b}) \; [\mathrm{YDS.\ IN.}^{-1}]$$

$$c_{\delta m} = -2.47 \pm 0.347 \qquad (r_{c_{\delta m}}) \; [\mathrm{YDS.\ OZ.}^{-1}]$$

$$c_\beta = -1434.01 \pm 313.48 \qquad (r_{c_\beta}) \; [\mathrm{YDS.\ IN.}^{-1}] .$$

The results c_β, $c_{\delta m}$, c_b and c_d when employed in the first residual equation (197) yield the following quantities:

Round Number	$c_\beta \beta$ [YDS.]	$c_b b$ [YDS.]	$c_{\delta m} \delta m$ [YDS.]	$c_d d$ [YDS.]	ξ_c [YDS.]	ξ_o [YDS.]	v_ξ [YDS.]
Standard							
2133	0.0	5.7	0.0	4.3	10.0	-- 23	33.0
2136		6.9		8.5	15.4	70	- 54.6
2139		3.4		-4.3	- 0.9	31	- 31.9
2142		5.7		4.3	10.0	22	- 12.0
2145		5.7		4.3	10.0	32	- 22.0
2148		3.4		8.5	11.9	- 38	49.9
2151		8.0		8.5	16.5	7	9.5
2154		8.0		12.8	20.8	- 34	54.8
2157		3.4		0.0	3.4	- 19	22.4
2160		5.7		0.0	5.7	90	- 84.3
2165		6.9		4.3	11.2	- 13	24.2
2170	0.0	9.2	0.0	17.1	26.3	- 78	104.3
Savanna							
2134	-64.5	0.0	0.0	- 4.3	- 68.8	- 34	- 34.8
2137	-57.4	0.0		8.5	- 48.9	9	- 57.9
2140	-67.4	-2.3		12.8	- 56.9	- 6	- 50.9
2143	-63.1	-1.1		12.8	- 51.4	3	- 54.4
2146	-43.0	2.3		- 4.3	- 45.0	- 87	42.0
2149	-43.0	1.1		8.5	- 33.4	- 58	24.6
2152	-61.7	-1.1		- 8.5	- 71.3	- 77	5.7
2155	-21.5	-2.3		0.0	- 23.8	- 23	- 0.8
2158	-43.0	-1.1		17.1	- 27.0	- 100	73.0
2161	-50.2	-1.1		17.1	- 34.2	- 21	- 13.2
2163	-47.3	-1.1		- 17.1	- 65.5	- 158	92.5
2166	-28.7	-1.1		0.0	- 29.8	- 105	75.2
2168	-33.0	-1.1		- 68.2	-102.3	- 56	- 46.3
2171	-28.7	-1.1		4.3	- 25.5	- 81	55.5
2173	-25.8	-2.3	0.0	8.5	- 19.6	- 26	6.4
Charleston							
2135	0.0	4.6	-152.9	0.0	-148.3	- 50	- 98.3
2138		-1.1	-143.1	- 21.3	-165.5	- 191	25.5
2141		-2.3	- 83.9	8.5	- 77.7	- 105	27.3
2144		2.3	- 83.9	- 12.8	- 94.4	21	-115.4
2147		-2.3	- 69.1	- 85.3	-156.7	- 194	37.3
2150		2.3	- 69.1	12.8	- 54.0	- 84	30.0
2153		6.9	- 66.6	- 89.5	-149.2	- 135	- 14.2
2156		0.0	- 66.6	- 8.5	- 75.1	- 191	115.9
2159		2.3	- 66.6	- 51.2	-115.5	- 159	43.5
2162		-1.1	- 64.1	4.3	- 60.9	- 135	74.1
2164		0.0	- 64.1	- 8.5	- 72.6	- 155	82.4
2167		0.0	- 64.1	- 34.1	- 98.2	- 680	581.8
2169		-1.1	- 64.1	- 8.5	- 73.7	- 34	- 39.7
2172		0.0	- 64.1	4.3	- 59.8	14	- 73.8
2174	0.0	-2.3	- 61.7	0.0	- 64.0	- 92	28.0

The quantities c_β, $c_{\delta m}$, c_b and c_d agree reasonably closely with the theoretical results obtained in previous sections and with the work of R. H. Kent[1] on the similar 155mm.Mk. III shell fired from the 155mm.Gun. The latter results are reduced to the present conditions by equivalent linear changes. Thus R. H. Kent found that a change in the band position forward (rearward) of a 0.05 [IN.] led to an effect of a + 0.24+% (-) in the range. Accordingly, at 9600 yards range, Mr. Kent would have:

$$c_b = 0.0024 \cdot 20 \cdot 9600 = 461 \quad \left[\text{YDS. IN.}^{-1}\right]. \tag{198}$$

On employing equation (117) to modify the result of $c_{\delta m}$ to the result for c_c, the value obtained from the present data is:

$$c_c = \begin{cases} -787.4 \pm 108.4 & \text{T.N.T.} \left[\text{YDS. IN.}^{-1}\right] \\ -800.0 \pm 110.1 & \text{50:50 AM.} \left[\text{YDS. IN.}^{-1}\right]. \end{cases} \tag{199}$$

Mr. Kent's results are:

c_r	1% Change in Range	Equivalent Effect at 9600 [YDS.]	Corresponding Value of c_c
0.01	-0.02	-1.92	-192
0.02	-0.13	-12.48	-624
0.03	-0.38	-36.48	-1216
0.04	-0.65	-62.40	-1560 .

It is perhaps desirable to make a direct comparison of the three results under the conditions prevailing for the shell examined in the present report. This comparison is afforded by the following short table:

		C RENO EXP.
c_β	$\left[\text{YDS. IN.}^{-1}\right]$	-1434.0 ± 313.5
	Mean $c = \bar{c}_{TNT} = 0.10173$ [IN.]	-787.4 ± 108.4 T.N.T.
c_c	Mean $c = \bar{c}_{50:50\ AM} = 0.10012$ [IN.]	-800.0 ± 11.0 50:50 AM
	YDS. IN.^{-1}	
	0.01	
	0.02	
	0.03	
	0.04	
$c_b \left[\text{YDS. IN.}^{-1}\right]$		114.5 ± 188.0
$c_d \left[\text{YDS. IN.}^{-1}\right]$		4263.3 ± 1139.5

[1] R. H. Kent: Memorandum Report on Tolerance Tests of 155mm·Projectiles O.P. No. 4084. On account of the dependence of the band position effect on other dimensions it is not certain that the quantity given by equation (198) is comparable with those given later for the shell examined in this report.

	C KENT EXP.
c [YDS. IN.$^{-1}$]	-1430.5 [1]
c_c Mean $c = \bar{c}_{TNT} = 0.10173$ [IN.]	
Mean $c = \bar{c}_{50:50\ AM} = 0.10012$ [IN.]	
[YDS. IN.$^{-1}$]	
0.01	-192 [2]
0.02	-624
0.03	-1216
0.04	-1560
c_b [YDS. IN.$^{-1}$]	461 [3]
c_d [YDS. IN.$^{-1}$]	2112 [4]

	C RENO THEORY
c [YDS. IN.$^{-1}$]	$+ 2900 \frac{\partial C}{C \partial_\beta}$
c_c Mean $c = \bar{c}_{TNT} = 0.10173$ [IN.]	$-171.3 + 1840.9 \frac{\partial C}{C \partial_r}$ TNT
Mean $c = \bar{c}_{50:50\ AM} = 0.10012$ [IN.]	$-174.0 + 1870.8 \frac{\partial C}{C \partial_r}$ (50:50 AM)
[YDS. IN.$^{-1}$]	
0.01	
0.02	
0.03	
0.04	
c_b [YDS. IN.$^{-1}$]	$-1.15 + 2900 \frac{\partial C}{C \partial_b}$
c_d [YDS. IN.$^{-1}$]	-515

1 Results with Mk. III Shell, $E = 30° 35'$, $v_o = 2410 \frac{FT}{SEC}$; 155mm.G.P.F. Gun Plot XI in R. H. Kent's Report cited previously.

2 Results with Mk. III Shell, $E = 30° 35'$, $v_o = 2410 \frac{FT}{SEC}$; 155mm.G.P.F. Gun Plot XIV in R. H. Kent's Report cited previously.

3 Results with Mk. III Shell, $E = 30° 35'$, $v_o = 2410 \frac{FT}{SEC}$; 155mm.G.P.F. Gun Plot V in R. H. Kent's Report cited previously.

4 Results with Mk. III Shell, $E = 30° 35'$, $v_o = 2410 \frac{FT}{SEC}$; 155mm.G.P.F. Gun Plot III in R. H. Kent's Report cited previously.

It is indicated that, for 9600 yards range with these shell:

$$\left.\begin{array}{llll}
\Delta_\beta X = -1434.0 \quad \beta & \dfrac{\Delta_\beta X}{X} = -0.1494 & \beta \\
\Delta_c X = -792.8 \quad c & \dfrac{\Delta_c X}{X} = -0.08258 & c \\
\Delta_b X = -114.5 \quad \varepsilon_b & \dfrac{\Delta_b X}{X} = +0.0\,1193 & \varepsilon_b \\
\Delta_d X = +\,4263.3 \quad \varepsilon_d & \dfrac{\Delta_d X}{X} = +0.4441 & \varepsilon_d\,.
\end{array}\right\} \quad (200)$$

To some extent the ratios given in the last column will be constant for conditions not far removed from those of the present report. These ratios may be employed to yield the range effects for other conditions without serious error.

The quantities k_β, $k_{\delta m}$, k_b and k_d are too small to have serious significance in respect to the effects in fire on the battlefield, and too small with respect to their probable errors to be reliable. These quantities are therefore not given. Without serious error, the augmentation of the variance in range may be computed by:

$$\left.\begin{array}{l}
\Delta_\beta \sigma_X^2 = c_\beta^2\, \Delta\sigma_\beta^2 = 2.056\cdot 10^6 \quad \Delta\sigma_\beta^2 \\
\Delta_c \sigma_X^2 = c_c^2\, \Delta\sigma_c^2 = 6.285\cdot 10^5 \quad \Delta\sigma_c^2 \\
\Delta_b \sigma_X^2 = c_b^2\, \Delta\sigma_b^2 = 1.311\cdot 10^4 \quad \Delta\sigma_b^2 \\
\Delta_d \sigma_X^2 = c_d^2\, \Delta\sigma_d^2 = 1.818\cdot 10^7 \quad \Delta\sigma_d^2\,.
\end{array}\right\} \quad (201)$$

The tolerances permissable for these shell may now be considered.

Figures 23 and 24 show the computed and indirectly observed results of the experimental firing with these shell in respect to the proportional range effects of eccentricity of boattail and mass to balance. The dispersion is considerable, but it appears that the foregoing comparison of theoretical and experimental values indicates their general reliabity. Then the following percentage effects may be expected:

β IN.	Percentage Range Effect $\frac{\Delta_\beta X}{X}$ %	Ratio of Range Effect to Probable Error of Individual Round [1] $\frac{\Delta_\beta X}{\varepsilon_{p.r.}}$
0.05	-0.747	-1.559
0,10	-1.494	-3.117
0.15	-2.241	-4.676
0.20	-2.988	-6.235
0.25	-3.735	-7.793

δm OZ.	Percentage Range Effect $\frac{\Delta_{\delta m} X}{X}$	Ratio of Range Effect to Probable Error of Individual Round $\frac{\Delta_{\delta m} X}{\varepsilon_{p.r.}}$
5	-0.128	-0.268
10	-0.257	-0.536
15	-0.358	-0.804
20	-0.514	-1.073
25	-0.642	-1.341
30	-0.771	-1.609
35	-0.899	-1.877
40	-1.028	-2.145
45	-1.156	-2.413
50	-1.285	-2.681
55	-1.413	-2.950
60	-1.542	-3.218
65	-1.670	-3.486
70	-1.799	-3.754
75	-1.927	-4.022

[1] The symbol $\varepsilon_{p.r.}$ denotes the firing table probable error in range, which at 9600 yards range for the present shell has the value 46 YDS. as given in F.T. 155-D-3.

ε_b	Percentage Range Effect	Ratio of Range Effect to Probable Error of Individual Round
IN.	$\frac{\Delta_b X}{X}$	$\frac{\Delta_b X}{\varepsilon_{p.r.}}$
0.05	0.060	0.124
0.10	0.119	0.249
0.15	0.179	0.373
0.20	0.239	0.498
0.25	0.298	0.622

ε_d	Percentage Range Effect	Ratio of Range Effect to Probable Error of Individual Round
IN.	$\frac{\Delta_d X}{X}$	$\frac{\Delta_d X}{\varepsilon_{p.r}}$
0.005	0.222	0.463
0.010	0.444	0.927
0.015	0.666	1.390
0.020	0.888	1.854
0.025	1.110	2.317

Let the greatest admissible proportional range effect or ratio of the range effect to the range probable error be selected in advance. Then the corresponding maximum allowable tolerance is given by the parallel value of the abnormality. In this way the maximum allowable tolerances from the standpoint of range effects may be obtained. A similar table would provide for determination of the maximum allowable tolerance with respect to the augmentation of dispersion in range.

C. Resumé of Numerical Results

It appears desirable to collect the numerical results obtained from the experiments described herein in a brief form in order to provide for rapid reference. Results obtained in this report for the 155mm Howitzer H.E. Shell, Mark I are compared with results obtained by R. H. Kent for the somewhat similar 155mm Gun H.E. Shell, Mark III when the data are reduced to the same firing conditions.

Abnormality	RENO Range Effect [YDS. IN.$^{-1}$]	Probable Error [YDS. IN.$^{-1}$]	KENT Range Effect [YDS. IN.$^{-1}$]
Eccentric Boattail	-1434	314	-1430
Eccentric Cavity			
0.01 [IN]			-192
0.02 [IN]			-624
0.03 [IN]			-1216
0.04 [IN]			-1560
~ 0.10 [IN]	-787 50:50 AM	108	
~ 0.10 [IN]	-800 T.N.T.	110	
Band Position	114	188	461
Diameter	4263	1140	

The effects on deflection of the abnormalitites given above are inconsequential.

The effects on the variance in range are significant.

Abnormality	Effect on Variance in Range
Eccentric Boattail	$2.06 \cdot 10^6$ Times Additional Variance in Eccentricity of Boattail
Eccentric Cavity	$6.28 \cdot 10^5$ Times Additional Variance in Eccentricity of Cavity
Band Position	$1.31 \cdot 10^4$ Times Additional Variance in Band Position
Diameter	$1.82 \cdot 10^7$ Times Additional Variance in Diameter.

For rapid computation of tolerances, it may be noted that the percentage range effect of

0.1 inch eccentricity of boattail is	1.5
10 ounces eccentric mass to balance is	0.26
0.1 inch error in band position is	0.12
0.01 inch error in bourrelet diameter is	0.44.

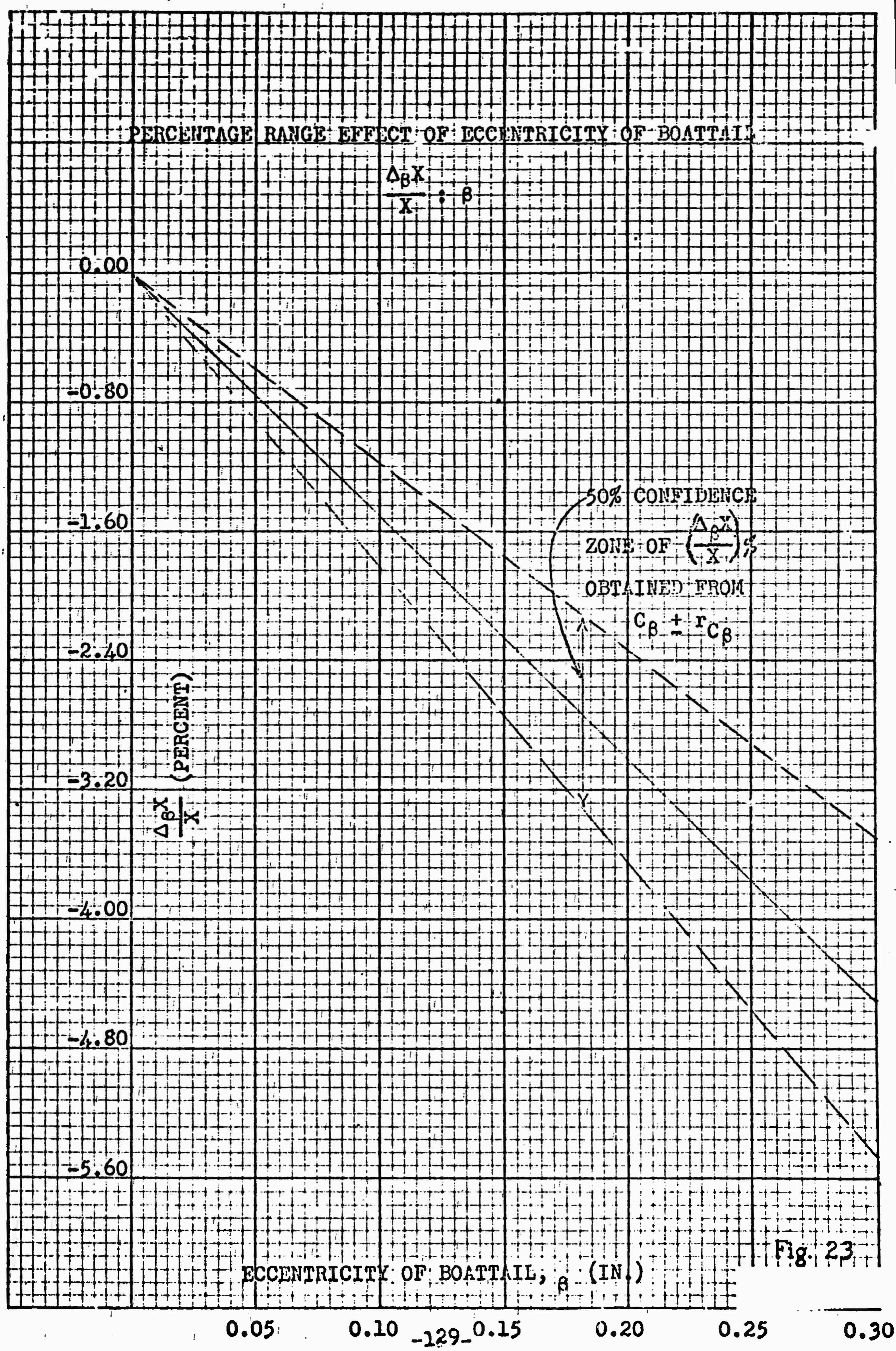
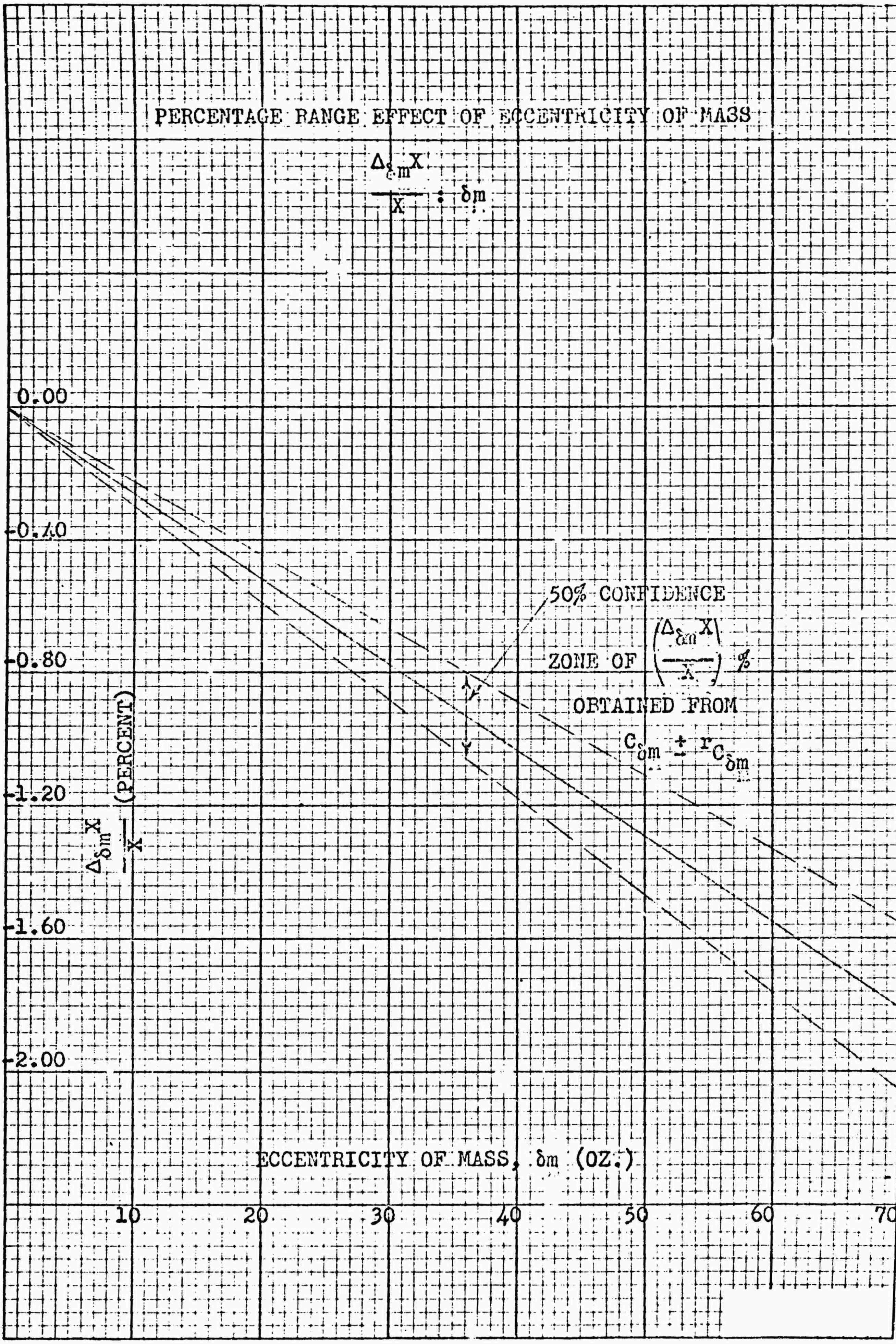

PERCENTAGE RANGE EFFECT OF ECCENTRICITY OF BOATTAIL
$\frac{\Delta_\beta X}{X} : \beta$
0.00
-0.80
-1.60
-2.40
-3.20
-4.00
-4.80
-5.60
$\frac{\Delta_\beta X}{X}$ (PERCENT)
50% CONFIDENCE ZONE OF $\left(\frac{\Delta_\beta X}{X}\right)$'S OBTAINED FROM $C_\beta \pm r_{C_\beta}$
ECCENTRICITY OF BOATTAIL, β (IN.)
0.05
0.10
0.15
0.20
0.25
0.30

Fig. 23



NO. 340-10 DIETZGEN GRAPH PAPER 10 PER INCH
EUGENE DIETZGEN CO. MADE IN U.S.A.

Fig. 24

The writer is indebted to Dr. L. S. Dederick for suggesting the general approach to the problem. There are incorporated in the report many general and specific suggestions kindly made after critical reading by Dr. E. J. McShane, Dr. J. L. Kelley and Major T. E. Sterne. Consultation with Mr. R. H. Kent on his work on the assignment of manufacturing tolerances has been of very great value.

F. V. Reno

F. V. Reno

APPENDIX A

GAUGE MEASUREMENTS OF
155MM HOWITZER H.E. SHELL MK.I WITH
M46 FUZE SHELL FROM
RANGE FIRING PROGRAM
FOR FIRING TABLE FT 155-D-3

N.O.D.

MEASUREMENT OF 155MM. SHELLS Lot 4408-1 DATE September 15, 1937

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
1	3.15"	6.081 6.081	22.805	6.197 6.192	.60"
2	3.15	6.081 6.081	22.805	6.195 6.195	.60
3	3.15	6.080 6.080	22.723	6.199 6.199	.60
4	3.15	6.080 6.081	22.828	6.196 6.198	.60
5	3.16	6.080 6.080	22.740	6.199 6.200	.60
6	3.17	6.079 6.079	22.649	6.199 6.199	.60
7	3.16 6.080	6.079 6.080	22.732	6.197 6.197	.60
8	3.17	6.075 6.078	22.732	6.196 6.196	.60
9	3.16	6.075 6.075	22.865	6.197 6.197	.60
10	3.17	6.080 6.080	22.725	6.201 6.201	.60
11	3.16	6.082 6.082	22.736	6.200 6.200	.60
12	3.16	6.079 6.079	22.800	6.201 6.199	.60
13	3.16	6.079 6.079	22.684	6.198 6.198	.60
14	3.16	6.076 6.079	22.760	6.202 6.198	.60
15	3.17	6.078 6.081	22.748	6.195 6.197	.60
16	3.17	6.081 6.081	22.7[illegible]0	6.198 6.201	.60
17	3.18	6.071 6.073	22.710	6.197 6.195	.60
18	3.18	6.079 6.079	22.710	6.194 6.196	.60
19	3.18	6.079 6.079	22.870	6.193 6.189	.60
20	3.17	6.079 6.079	22.815	6.197 6.196	.60
21	3.17	6.079 6.079	22.740	6.193 6.193	.60
22	3.17	6.079 6.079	22.885	6.200 6.195	.60
23	3.18	6.068 6.069	22.856	6.190 6.191	.60
24	3.17	6.078 6.079	22.857	6.196 6.196	.60
25	3.18	6.076 6.076	22.628	6.196 6.198	.60
26	3.16	6.076 6.080	22.721	6.193 6.191	.60
27	3.18	6.075 6.075	22.672	6.193 6.190	.60

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
28	3.17	6.082 6.082	22.754	6.195 6.195	.60
29	3.18	6.078 6.078	22.754	6.203 6.200	.60
30	3.18	6.081 6.083	22.783	6.195 6.195	.60
31	3.17	6.078 6.079	22.827	6.186 6.186	.60
32	3.17	6.082 6.082	22.780	6.198 6.202	.60
33	3.17	6.081 6.083	22.780	6.199 6.200	.60
34	3.18	6.079 6.079	22.798	6.194 6.194	.60
35	3.17	6.076 6.078	22.773	6.195 6.197	.60
36	3.16	6.080 6.080	22.760	6.197 6.197	.60
37	3.17	6.075 6.077	22.740	6.196 6.198	.60
38	3.18	6.081 6.081	22.757	6.199 6.202	.60
39	3.17	6.075 6.075	22.736	6.198 6.200	.60
40	3.17	6.081 6.083	22.736	6.200 6.200	.60
41	3.17	6.077 6.077	22.723	6.195 6.198	.60
42	3.16	6.077 6.079	22.697	6.187 6.189	.61
43	3.17	6.075 6.077	22.748	6.195 6.195	.60
44	3.18	6.077 6.082	22.727	6.195 6.200	.61
45	3.17	6.081 6.081	22.740	6.200 6.200	.60
46	3.15	6.070 6.076	22.704	6.196 6.196	.60
47	3.16	6.079 6.080	22.687	6.194 6.194	.60
48	3.16	6.080 6.080	22.719	6.198 6.198	.60
49	3.15	6.080 6.082	22.727	6.200 6.201	.60
50	3.16	6.079 6.076	22.828	6.193 6.193	.60
51	3.16	6.076 6.077	22.705	6.197 6.197	.60
52	3.16	6.077 6.078	22.750	6.195 6.195	.60
53	3.17	6.074 6.074	22.720	6.190 6.195	.60
54	3.16	6.079 6.079	22.747	6.198 6.198	.61

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
55	3.17	6.074	22.706	6.181	.59
		6.074		6.181	
56	3.16	6.078	22.720	6.182	.60
		6.078		6.182	
57	3.16	6.079	22.797	6.197	.60
		6.081		6.193	
58	3.16	6.080	22.727	6.197	.60
		6.080		6.199	
59	3.17	6.075	22.727	6.195	.60
		6.075		6.195	
60	3.17	6.063	22.800	6.195	.61
		6.068		6.196	
61	3.17	6.069	22.668	6.191	.60
		6.069		6.196	
62	3.17	6.078	22.740	6.197	.60
		6.080		6.197	
63	3.17	6.080	22.747	6.188	.60
		6.080		6.190	
64	3.17	6.078	22.820	6.194	.60
		6.081		6.198	
65	3.15	6.078	22.743	6.195	.60
		6.077		6.196	
66	3.17	6.080	22.743	6.194	.60
		6.080		6.194	
67	3.18	6.079	22.697	6.194	.60
		6.077		6.190	
68	3.18	6.072	22.803	6.186	.60
		6.074		6.186	
69	3.18	6.077	22.727	6.193	.60
		6.077		6.195	
70	3.17	6.080	22.727	6.198	.60
		6.080		6.198	
71	3.15	6.079	22.727	6.198	.60
		6.079		6.198	
72	3.17	6.069	22.674	6.195	.60
		6.069		6.195	
73	3.17	6.080	22.692	6.194	.60
		6.080		6.194	
74	3.18	6.072	22.778	6.193	.60
		6.077		6.193	
75	3.[illegible]	6.079	22.704	6.190	.60
		6.079		6.194	
76	3.17	6.081	22.683	6.190	.60
		6.081		6.190	
77	3.18	6.078	22.765	6.195	.60
		6.075		6.195	
78	3.17	6.081	22.697	6.193	.60
		6.081		6.193	
79	3.16	6.077	22.710	6.197	.60
		6.077		6.197	
80	3.18	6.078	22.692	6.193	.60
		6.078		6.194	

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
81	3.17	6.080, 6.080	22.692	6.198, 6.198	.60
82	3.17	6.080, 6.080	22.796	6.200, 6.200	.60
83	3.17	6.080, 6.080	22.760	6.197, 6.202	.60
84	3.16	6.082, 6.083	22.744	6.196, 6.199	.60
85	3.16	6.078, 6.078	22.667	6.195, 6.195	.60
86	3.17	6.073, 6.075	22.729	6.199, 6.201	.60
87	3.17	6.080, 6.080	22.775	6.191, 6.192	.60
88	3.17	6.073, 6.075	22.815	6.195, 6.195	.60
89	3.18	6.073, 6.076	22.762	6.198, 6.198	.60
90	3.17	6.078, 6.079	22.652	6.192, 6.192	.60
91	3.21	6.081, 6.082	22.670	6.195, 6.195	.60
92	3.19	6.079, 6.080	22.702	6.196, 6.196	.60
93	3.15	6.079, 6.079	22.730	6.195, 6.195	.60
94	3.19	6.082, 6.082	22.803	6.196, 6.196	.60
95	3.19	6.081, 6.081	22.715	6.198, 6.198	.60
96	3.18	6.081, 6.081	22.740	6.194, 6.199	.60
97	3.18	6.081, 6.081	22.725	6.197, 6.200	.60
98	3.17	6.077, 6.076	22.845	6.197, 6.197	.60
99	3.17	6.078, 6.078	22.832	6.199, 6.201	.60
100	3.18	6.078, 6.078	22.756	6.194, 6.194	.60
101	3.18	6.079, 6.079	22.772	6.195, 6.203	.60
102	3.18	6.078, 6.080	22.796	6.196, 6.197	.60
103	3.15	6.074, 6.075	22.887	6.190, 6.193	.60
104	3.15	6.078, 6.078	22.757	6.195, 6.195	.60
105	3.15	6.078, 6.080	22.631	6.193, 6.195	.60
106	3.18	6.081, 6.081	22.710	6.193, 6.186	.60

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
107	3.18	6.080	22.737	6.199	.60
		6.080		6.199	
108	3.17	6.067	22.820	6.199	.60
		6.067		6.199	
109	3.18	6.073	22.743	6.193	.60
		6.073		6.196	
110	3.17	6.080	22.705	6.196	.60
		6.082		6.197	
111	3.17	6.081	22.662	6.197	.60
		6.080		6.197	
112	3.17	6.080	22.815	6.195	.60
		6.080		6.195	
113	3.17	6.076	22.698	6.198	.60
		6.078		6.198	
114	3.17	6.080	22.745	6.192	.60
		6.078		6.194	
115	3.17	6.078	22.775	6.193	.60
		6.078		6.195	
116	3.18	6.075	22.756	6.193	.60
		6.080		6.194	
117	3.17	6.079	22.870	6.198	.60
		6.079		6.198	
118	3.18	6.081	22.705	6.193	.60
		6.081		6.193	
119	3.17	6.077	22.785	6.193	.60
		6.077		6.194	
120	3.18	6.081	22.720	6.196	.60
		6.081		6.197	
121	3.19	6.065	22.770	6.200	.60
		6.075		6.202	
122	3.19	6.080	22.765	6.194	.60
		6.079		6.194	
123	3.17	6.082	22,760	6.199	.60
		6.082		6.199	
124	3.17	6.078	22.760	6.198	.60
		6.082		6.198	
125	3.20	6.062	22.850	6.194	.60
		6.067		6.194	
126	3.18	6.080	22.730	6.199	.60
		6.080		6.199	
127	3.16	6.081	22.825	6.199	.60
		6.081		6.199	
128	3.21	6.072	22.868	6.196	.60
		6.079		6.197	
129	3.16	6.080	22,745	6.196	.60
		6.079		6.200	
130	3.19	6.082	22.775	6.200	.60
		6.079		6.200	
131	3.16	6.081	22.735	6.198	.60
		6.081		6.198	
132	3.18	6.075	22.687	6.196	.60
		6.075		6.196	

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
133	3.18	6.074 6.076	22.778	6.195 6.196	.60
134	3.19	6.076 6.076	22.740	6.197 6.198	.60
135	3.18	6.076 6.076	22.820	6.196 6.196	.60
136	3.17	6.074 6.074	22.804	6.203 6.201	.60
137	3.17	6.082 6.080	22.770	6.193 6.196	.60
138	3.17	6.080 6.080	22.770	6.196 6.196	.60
139	3.16	6.080 6.080	22.796	6.196 6.196	.60
140	3.17	6.073 6.073	22.815	6.199 6.199	.60
141	3.17	6.077 6.077	22.784	6.198 6.198	.60
142	3.17	6.081 6.083	22.807	6.202 6.196	.60
143	3.17	6.075 6.076	22.807	6.192 6.196	.60
144	3.17	6.076 6.076	22.860	6.188 6.190	.60
145	3.18	6.075 6.081	22.722	6.194 6.195	.60
146	3.18	6.079 6.079	22.808	6.197 6.199	.60
147	3.18	6.077 6.078	22.755	6.199 6.202	.60
148	3.18	6.078 6.078	22.865	6.199 6.201	.60
149	3.18	6.078 6.078	22.735	6.196 6.197	.60
150	3.17	6.078 6.078	22.840	6.197 6.197	.60
151	3.16	6.072 6.072	22.748	6.187 6.190	.60
152	3.16	6.080 6.082	22.715	6.190 6.197	.60
153	3.18	6.080 6.080	22.740	6.200 6.200	.60
154	3.18	6.080 6.080	22.780	6.198 6.196	.60
155	3.18	6.078 6.078	22.694	6.187 6.190	.60
156	3.15	6.076 6.077	22.820	6.180 6.185	.60
157	3.17	6.080 6.082	22.760	6.196 6.196	.60
158	3.16	6.070 6.071	22.760	6.196 6.196	.60

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
159	3.17	6.080 6.080	22.715	6.194 6.194	.60
160	3.16	6.077 6.080	22.749	6.193 6.196	.60
161	3.17	6.080 6.082	22.830	6.195 6.195	.60
162	3.16	6.081 6.081	22.722	6.200 6.199	.60
163	3.16	6.078 6.080	22.732	6.195 6.199	.60
164	3.17	6.077 6.079	22.790	6.195 6.195	.60
165	3.20	6.077 6.079	22.762	6.196 6.196	.60
166	3.18	6.080 6.080	22.796	6.195 6.195	.60
167	3.18	6.081 6.081	22.900	6.197 6.197	.60
168	3.17	6.081 6.081	22.852	6.197 6.197	.60
169	3.17	6.073 6.075	22.785	6.183 6.184	.60
170	3.18	6.080 6.082	22.695	6.187 6.187	.60
171	3.19	6.081 6.081	22.764	6.197 6.200	.60
172	3.18	6.078 6.078	22.822	6.193 6.195	.60
173	3.17	6.073 6.075	22.810	6.194 6.196	.60
174	3.17	6.080 6.080	22.712	6.197 6.197	.60
175	3.17	6.078 6.080	22.858	6.195 6.192	.60
176	3.17	6.076 6.074	22.828	6.193 6.194	.60
177	3.16	6.078 6.078	22.815	6.198 6.198	.60
178	3.18	6.079 6.082	22.740	6.195 6.193	.60
179	3.17	6.081 6.081	22.762	6.194 6.195	.60
180	3.17	6.081 6.081	22.750	6.198 6.198	.60
181	3.17	6.074 6.074	22.832	6.197 6.195	.60
182	3.17	6.080 6.080	22.765	6.204 6.203	.60
183	3.17	6.077 6.077	22.792	6.196 6.196	.60
184	3.17	6.078 6.079	22.735	6.198 6.198	.60

Shell No.	Distance Base to Band	Diam. of Bourrelet	Total Length of Shell	Diam. of Band	Width of Band
185	3.17	6.080 6.080	22.735	6.193 6.194	.60
186	3.17	6.074 6.076	22.850	6.188 6.186	.60
187	3.17	6.076 6.076	22.700	6.193 6.195	.60
188	3.17	6.081 6.081	22.740	6.192 6.189	.60
189	3.17	6.080 6.080	22.765	6.193 6.193	.60
190	3.16	6.078 6.078	22.736	6.194 6.194	.60
191	3.16	6.081 6.081	22.705	6.190 6.193	.60
192	3.10	6.081 6.079	22.784	6.196 6.196	.60
193	3.16	6.078 6.080	22.755	6.199 6.203	.60
194	3.14	6.080 6.080	22.818	6.195 6.198	.60
195	3.18	6.073 6.073	22.725	6.197 6.200	.60
196	3.17	6.078 6.078	22.784	6.195 6.195	.60
197	3.18	6.074 6.075	22.742	6.198 6.198	.60
198	3.18	6.078 6.078	22.816	6.195 6.196	.60
199	3.17	6.080 6.080	22.742	6.199 6.199	.60
200	3.18	6.082 6.082	22.802	6.194 6.194	.60

Measured for Major Hofstetter

Measured by R. L. Greenland - C. Kotras

Clerk - A. G. McConnell

Date - 9/16/37 Time - 10:45 A.M.

W.O. - 107-1

APPENDIX B

FIRING RECORD 11004
RANGE FIRING TO
DETERMINE EFFECT
OF ECCENTRICITY IN
WEIGHT AND OF
BOATTAIL OF
155MM.HOWITZER H.E. SHELL MK.I

ABERDEEN PROVING GROUND FIRINGS

Object of Firing: Range Firing to Determine Effect of Eccentricity in Weight and of Boat tail of 155 M/M Howitzer Shell Mk. I

DEVELOPMENT

Date of Firing May 3, 1930
Firing Record No. 11004
Sheet 1 of 6
T. S. T. P.
O. C. M. Item
O. P. No. 4703
Contract No.
O. O. File 471.16/2245
A. P. G. File 471.151/142
W. O. No. 326-1 11

Related F. R. Nos. 11005

	CALIBER	MODEL	MANUFACTURER	No.	ROUNDS FIRED PRIOR TO TEST
Cannon	155 M/M Howitzer	1918	A.B.S. & F. Co.	855	2131
Carriage	155 M/M Howitzer	1918	Standard Steel Car Co.	129	
Recoil Mech	155 M/M Howitzer	1918	Dodge Bros.	145	

Azimuth of line of fire | Deflection from AP Mils
Gun position S-4 Right Bay | Target

Projectile	Mk. 1 Slug (Rd. 2132) 155 M/M Mk. I Shell for Howitzer, S.O.D. Ammunition Lot Nos. 7884-1, 5291-1, -4, -5, -26, & -27 and Special lot from C.O.D. (For all other rounds)
Bursting charge	50/50 Amatol for Shell Ammunition Lot Nos. 7884-1, 5291-26 & -27. T.N.T. for all other lots.
Booster	Mk. IIIA MII for Amm. Lot 7884-1 Mk. IIIA MI for all other lots.
Fuze	Point Detonating, M46, P.A. Ammunition Lot No. 5938-2. Fuzes set superquick.
Powder	D.P. Pyro Lot X-1495-1918 for 155 M/M Howitzer, M1917
~~Grenade~~ bag	Single Section
Igniter	3 ozs. Black Powder in base of single section
Primer	21 Gr. Perc. Mk. 11A, Federal Lot 1117-31, Dwg. 74-2-21

ABERDEEN PROVING GROUND FIRINGS

GENERAL DATA BY ROUNDS

1938 DATE May	ROUND NO.	TIME OF FIRING	PROJECTILE: APG No. XL	PROJECTILE: Weight as Fired lbs. ozs.	PROJECTILE: Seating Ins.	POWDER: Lot	POWDER: Box No.	POWDER: Charge Weight lbs. ozs.	ELEVATION Deg. Min.	FINAL CORRECTED: Pressure	FINAL CORRECTED: Scale Velocity	Shell Ammunition Lot
3	2132	9:13		96 0	20-7/16	X-1495		5 0	23 15		1105	
	2133	9:44	2-S-7	96 10	19-3/4	"		7 6.4	" "	28500	1470	78[illegible]-
	2134	9:51	2-S-67	95 6½	20	"		" "	" "	27100	1461	5291-
	2135	9:55	2-S-43	94 11	19-15/16	"		" "	" "	27900	1464	Spec
	2136	10:07	2-S-6	96 10	19-15/16	"		" "	" "	28100	1461	78[illegible]
	2137	10:13	2-S-66	95 5¼	19-15/16	"		" "	" "	28500	1469	5291-
	2138	10:17	2-S-46	95 2	19-15/16	"		" "	" "	27500	1469	Spec
	2139	10:19	2-S-5	96 10	19-7/8	"		" "	" "	25700	1462	7884-
	2140	10:29	2-S-59	95 3	20-1/8	"		" "	" "	28100	1453	5291-
	2141	10:31	2-S-42	94 6	19-15/16	"		" "	" "	27700	1470	Spec
	2142	10:33	2-S-4	96 10	20	"		" "	" "	27000	1467	78[illegible]
	2143	10:39	2-S-[illegible]	95 [illegible]	20	"		" "	" "	27200	1472	5291
	2144	10:45	2-S-[illegible]	93 [illegible]	20	"		" "	" "	27400	1455	Spec
	2145	10:48	2-S-[illegible]	95 0	19-11/16	"		" "	" "	28100	1464	7[illegible]
	2146	10:51	2-S-[illegible]	94 7	20	"		" "	" "	27500	1466	5291-
	2147	10:53	2-S-45	95 10	19-15/16	"		" "	" "	27000	1460	[illegible]
	2148	10:55	2-S-2	96 8	20	"		" "	" "	29200	1466	7[illegible]
	2149	11:05	2-S-54	93 15	20	"		" "	" "	27600	1457	5291-
	2150	11:08	2-S-49	93 9	20	"		" "	" "	27400	1456	Spec
	2151	11:10	2-S-12	96 8	19-15/16	"		" "	" "	28100	1455	7[illegible]
	2152	11:12	2-S-65	94 5½	20	"		" "	" "	29100	1468	5291-
	2153	11:14	2-S-41	93 1	19-15/16	"		" "	" "	27500	1456	Spec
	2154	11:27	2-S-11	93 8	20	"		" "	" "	27400	1460	7[illegible]
	2155	11:30	2-S-51	94 8	20	"		" "	" "	27500	1456	5291-
	2156	11:32	2-S-44	93 2½	20	"		" "	" "	27200	1456	Spec
	2157	11:34	2-S-8	96 8	19-15/16	"		" "	" "	28700	1462	78[illegible]-
	2158	11:37	2-S-60	93 15	20	"		" "	" "	28100	1444	5291-
	2159	11:50	2-S-[illegible]	93 10	20	"		" "	" "	27500	1458	Spec
	2160	11:52	2-S-10	96 8	20-1/16	"		" "	" "	28100	1447	78[illegible]-
	2161	11:54	2-S-6[illegible]	94 13½	20	"		" "	" "	28200	1455	5291-
	2162	11:57	2-S-[illegible]	94 1	20	"		" "	" "	28100	1462	Spec
	2163	12:01	2-S-53	93 2-[illegible]	20	"		" "	" "	27000	1469	5291-
	2164	12:54	2-S-32	93 3	20	"		" "	" "	27100	1472	Spec
	2165	12:59	2-S-9	96 8	20	"		" "	" "	29700	1471	7[illegible]
	2166	1:02	2-S-55	9[illegible] [illegible]	19-15/16	"		" "	" "	28100	1456	5291-
	2167	1:05	2-S-5[illegible]	9[illegible] [illegible]	20	"		" "	" "	26600	1453	Spec
	2168	1:07	2-S-[illegible]2	94 5	20	"		" "	" "	27500	1470	5291-
	2169	1:10	2-S-40	93 10	20	"		" "	" "	27000	1473	Spec
	2170	1:23	2-S-15	96 8	19-15/16	"		" "	" "	28000	1462	78[illegible]-
	2171	1:25	2-S-6[illegible]	94 6	20	"		" "	" "	27200	1473	5291-
	2172	1:27	2-S-3[illegible]	93 1	19-15/16	"		" "	" "	27200	1457	Spec
	2173	1:29	2-S-57	94 [illegible]	20	"		" "	" "	27200	1475	5291-
	2174	1:32	2-S-5[illegible]	94 5	19-15/16					28000	1456	Spec

Rd. 2132, warming round.

Temperature of powder = 71°F

ABERDEEN PROVING GROUND FIRINGS

VELOCITY DATA

Cannon 155 M/M How. M1918, No. 855 Fired by Maj. C. F. Hofstetter on May 3, 1923

Screen Distances	Gun to First	Horizontal	Corrected to	Between	Horizontal	Corrected to
	Coil	51.65 ft.		Coil	15.015 ft.	
	Screen			Screen		

Round No.	Time of Firing	Form Factor	Boulengé: Chronograph Number			Boulengé: Mean Instrumental	Boulengé: Muzzle Velocity	Solenoid: Instrumental	Solenoid: Muzzle Velocity
2132	9:35	1 = 1.78						1100	[illegible]
2133	9:44	1 = .72						1465	1470
2134	9:51							1456	1461
2135	9:55							1459	[illegible]
2136	10:07							1455	1461
2137	10:15							1454	1469
2138	10:27							1444	1469
2139	10:30							1457	[illegible]
2140	[illegible]							1458	[illegible]
2141	[illegible]							1465	[illegible]
2142	[illegible]							1462	[illegible]
2143	[illegible]							1453	[illegible]
2144	10:45							1460	[illegible]
2145	10:48							1459	[illegible]
2146	10:51							1462	[illegible]
2147	10:55							1464	1469
2148	10:57							1461	[illegible]
2149	11:07							1452	1457
2150	11:08							1451	[illegible]
2151	11:10							1450	1455
2152	11:12							1463	[illegible]
2153	11:14							1451	1456
2154	11:27							1455	[illegible]
2155	11:30							1451	[illegible]
2156	11:32							1461	1465
2157	11:34							1457	1462
2158	11:37							1469	1474
2159	11:50							1453	[illegible]
2160	11:52							1442	1447
2161	11:54							1450	[illegible]
2162	11:57							1467	1472
2163	12:01							1464	[illegible]
2164	12:54							1467	1472
2165	12:59							1466	1471
2166	1:02							1461	[illegible]
2167	1:05							1455	[illegible]
2168	1:07							1459	[illegible]
2169	1.10							1460	[illegible]
2170	1:23							1455	[illegible]
2171	1:25							1463	[illegible]
2172	1:27							1452	[illegible]
2173	1:29							1448	[illegible]
2174	1:32							1461	[illegible]

*This correction from Instrumental Velocity to Muzzle velocity includes a Tuning Fork correction.

ABERDEEN PROVING GROUND FIRINGS

F. R. No. 11004
Sheet 4 of 6
Date May 3, 1933

PRESSURE DATA

Type of gauge Medium Caliber
Position of gauge At rear of charge
Metal of crusher cylinder Sept. 12, 1918. Annealed April 4, 1919.
Initial compression 0

ROUND NO.	BAND DIAM. INS.	GAUGE NO.	PRESSURE / 100	GAUGE NO.	PRESSURE / 100	GAUGE NO.	PRESSURE / 100	GAUGE NO.	PRESSURE / 100	MEAN
2133		4203	280	5371	289					285
2134		4452	279	4614	263					271
2135		5540	273	4039	284					279
2136		4102	288	2925	273					281
2137		4497	280	5742	285					283
2138		2888	270	4447	280					275
2139		5993	292	[illegible]	281					287
2140		5696	280	4530	282					281
2141		5944	284	4537	269					277
2142		3077	278	4935	280					279
2143		2912	266	1712	280					273
2144		5926	274	[illegible]	276					275
2145		4646	289	2455	288					289
2146		4955	272	5786	277					275
2147		6042	278	4786	278					278
2148		5755	294	5671	290					292
2149		5602	278	4773	274					276
2150		2721	276	5727	271					274
2151		3237	277	5949	284					281
2152		3768	282	4311	280					281
2153		3230	274	6036	276					275
2154		4525	280	2246	268					274
2155		4928	275	4644	275					275
2156		4356	267	4112	276					272
2157		5110	286	4039	291					289
2158		4856	289	5787	273					281
2159		1744	274	4047	276					275
2160		3621	287	4774	274					281
2161		4193	278	5927	282					280
2162		4378	282	2325	280					281
2163		4803	Slid	4612	272					272
2164		5125	277	4153	276					277
2165		4579	297	5947	296					297
2166		4160	285	2422	276					281
2167		5949	277	4210	255					266
2168		4857	275	4118	275					275
2169		4375	277	1311	275					276
2170		5399	287	4822	292					290
2171		4602	281	[illegible]	276					279
2172		4920	265	4320	272					279
2173		[illegible]	270	4276	276					273
2174		[illegible]	282	[illegible]	277					280

Pressures in this report are read and calculated to the nearest one hundred lbs.

ABERDEEN PROVING GROUND FIRINGS

P.R. No. 11004
Sheet 5 of 6
Date May 3, 1953

MISCELLANEOUS DATA

Rd. No.	Shell Identification Serial No.	Zone	Lot No.	Crater Measurements - Inches Length	Width	Depth	Range Yards	Defl. Yards
2133	1	6	7884-1	72	84	19	9683	96
2134	1S	5	5291-1	72	84	25	9573	78
2135	1C	3	Special	In shell hole			9556	81
2136	2	6	7884-1	96	96	19	9715	88
2137	2S	5	5291-1	71	82	22	9663	79
2138	2C	4	Special	72	73	18	9459	105
2139	3	6	7884-1	In shell hole			9682	84
2140	3S	5	5291-1	60	72	16	9535	86
2141	3C	3	Special	In shell hole			9518	97
2142	4	6	7884-1	74	82	22	9706	88
2143	4S	5	5291-1	70	81	21	9579	86
2144	4C	3	Special	96	94	18	9604	81
2145	5	4	7884-1	74	82	32	9646	90
2146	5S	4	5291-4	84	84	20	9524	80
2147	5C	3	Special	75	76	14	9468	88
2148	6	6	7884-1	71	82	22	9636	87
2149	6S	4	5291-4	In shell hole			9472	85
2150	6C	3	Special	72	74	20	9433	91
2151	7	6	7884-1	71	74	19	9608	89
2152	7S	4	5291-4	72	84	25	9542	76
2153	7C	2	Special	72	72	25	9367	82
2154	8	6	7884-1	74	86	22	9599	89
2155	8S	4	5291-4	72	84	21	9520	77
2156	8C	2	Special	79	92	30	9373	82
2157	9	6	7884-1	96	94	24	9625	87
2158	9S	4	5291-4	72	72	20	9540	80
2159	9C	3	Special	72	74	20	9365	87
2160	10	6	7884-1	In old hole			9630	70
2161	10S	5	5291-5	70	72	18	9520	66
2162	10C	3	Special	70	70	20	9497	83
2163	11S	3	5291-26	84	78	29	9423	82
2164	11C	2	Special	84	96	24	9330	63
2165	11	6	7884-1	60	71	19	9615	87
2166	12S	4	5291-27	84	99	23	9415	68
2167	12C	3	Special	In water			8779	77
2168	13S	4	5291-27	72	72	20	9497	55
2169	13C	3	Special	70	71	15	9521	58
2170	12	6	7884-1	In old hole			9487	83
2171	14S	4	5291-27	72	72	22	9501	69
2172	14C	4	Special	72	72	20	9513	63
2173	15S	4	5291-27	72	74	21	9423	66
2174	15C	3	Special	In old hole			9442	63

Shells serially numbered "S" are Savanna lots. Those numbered "C" are Charleston lots. Those with no letter after serial number are shell from Savanna received for acceptance test and fired for comparison with the eccentric shell.

ABERDEEN PROVING GROUND FIRINGS

F. R. No. 11004
Sheet 5A of 6
Date

MISCELLANEOUS DATA

RD.NO.	WT. OF PROJ. LBS.	WT. OF PROJ. OZS.	MUZZLE VELOCITY	M.V. COR. FOR WT. OF PROJ.	RANGE YARDS	RANGE CORRECTED FOR WT. OF PROJECTILE
2133	96	10	1470	1477	9683	9693
2134	95	6½	1461	1463	9573	9578
2135	94	11	1464	1463	9556	9557
2136	96	10	1461	1468	9715	9724
2137	95	5½	1469	1470	9663	9668
2138	95	2	1469	1470	9459	9464
2139	96	10	1462	1469	9682	9691
2140	95	½	1453	1453	9535	9540
2141	94	0	1470	1466	9518	9518
2142	96	10	1467	1474	9706	9715
2143	95	2½	1458	1459	9579	9584
2144	93	13½	1465	1460	9604	9601
2145	95	0	1464	1464	9646	9651
2146	94	7½	1466	1464	9524	9524
2147	95	10½	1463	1471	9468	9473
2148	96	8	1466	1473	9636	9645
2149	93	13	1457	1452	9472	9469
2150	93	9	1456	1450	9433	9428
2151	96	8	1455	1462	9608	9618
2152	94	5½	1463	1465	9542	9542
2153	93	1	1456	1448	9367	9360
2154	96	8	1460	1467	9599	9609
2155	94	8	1456	1454	9520	9520
2156	93	2½	1465	1461	9379	9372
2157	96	8	1462	1469	9625	9634
2158	93	13	1474	1469	9540	9538
2159	93	10	1458	1452	9365	9362
2160	96	8	1447	1454	9630	9639
2161	94	13½	1455	1453	9520	9524
2162	94	1	1472	1468	9497	9497
2163	93	2-3/4	1469	1464	9423	9417
2164	93	3	1472	1467	9380	9373
2165	96	8	1471	1478	9615	9625
2166	94	½	1466	1462	9415	9415
2167	93	13½	1458	1453	8779	8771
2168	94	5	1470	1467	9497	9497
2169	93	10½	1473	1467	9521	9517
2170	96	8	1461	1468	9487	9497
2171	94	6	1473	1470	9501	9501
2172	95	1	1457	1457	9513	9517
2173	94	1½	1453	1450	9423	9423
2174	94	3	1466	1462	9442	9442

	Mean Range	Mean Range when cor. for wt. of proj.
Standard Shell	9556	[illegible]
Lowman Lot, Eccentric, Boat tail	9515	9516
Charleston Lots, Eccentric Wt.	9464	9463

MISCELLANEOUS DATA

No change in howitzer or carriage since last firing.

There were no hangfires, misfires, flarebacks or evidence of unconsumed powder on any round.

Howitzer and carriage functioned satisfactorily.

Thin amount of light gray smoke and medium yellow flash on all rounds.

All rounds detonated with high order and superquick action on impact.

Bore sight elevation = 3° 04'.

Gun bore-sighted for each round.

Azimuth line of fire = 36° 27'.

Cant of axle before Rd. 2133. Right trunnion above left = .009 ft. After firing, right trunnion above left = .012 ft. Distance between points = 2.14 ft.

Rd. 2167 not located, struck short. Estimated 800 yards short - functioned high order. This round was heard to whistle quite loudly and unevenly on firing, indicating erratic flight. However most of the "C" shell (those eccentric in weight) gave this uneven whistling sound to considerable degree.

This howitzer clinometered by Gauge Section against quadrant employed in firing this program.

Clinometer reading = 23° 15'.
Quadrant reading = 23° 15'

No correction required.

Shell of Lot 7884-1 which were fired for comparison of flight characteristices were from ballistic sample for acceptance and are included in report of acceptance test on Firing Record No. 11005.

~~[illegible]~~.

See attached sheets for all special measurements of eccentric shell.

Attached hereto is sheet showing tabulation of corrected velocities and ranges.

C. F. Hofstetter

C. F. HOFSTETTER,
Major, Ord. Dept.,
Proof Officer.

K. F. Adamson

K. F. A[illegible],
Lt.Col., Ord.Dept.,
Chief Proof Officer,
Gun Testing Division.

APPROVED:

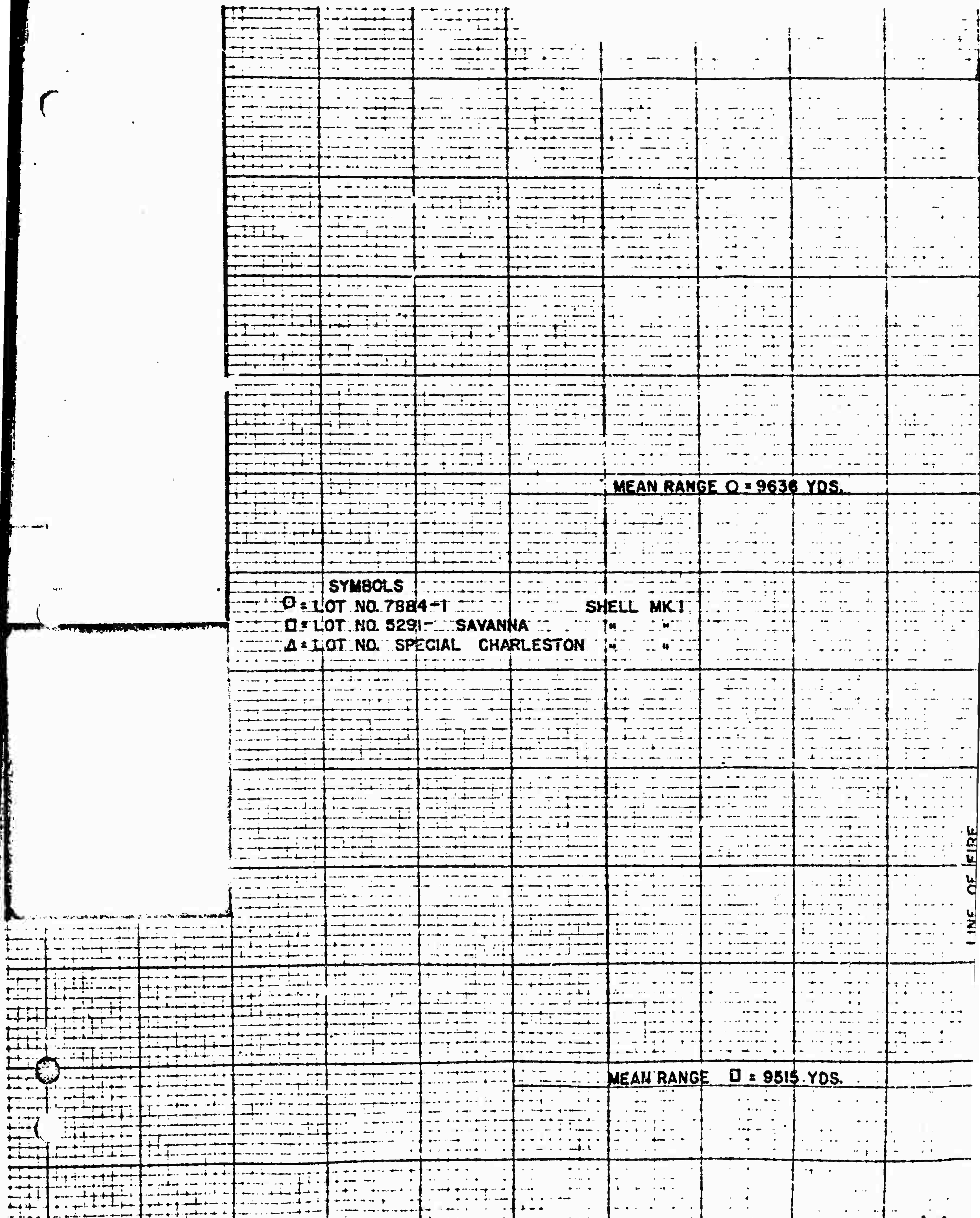
K. F. Adamson

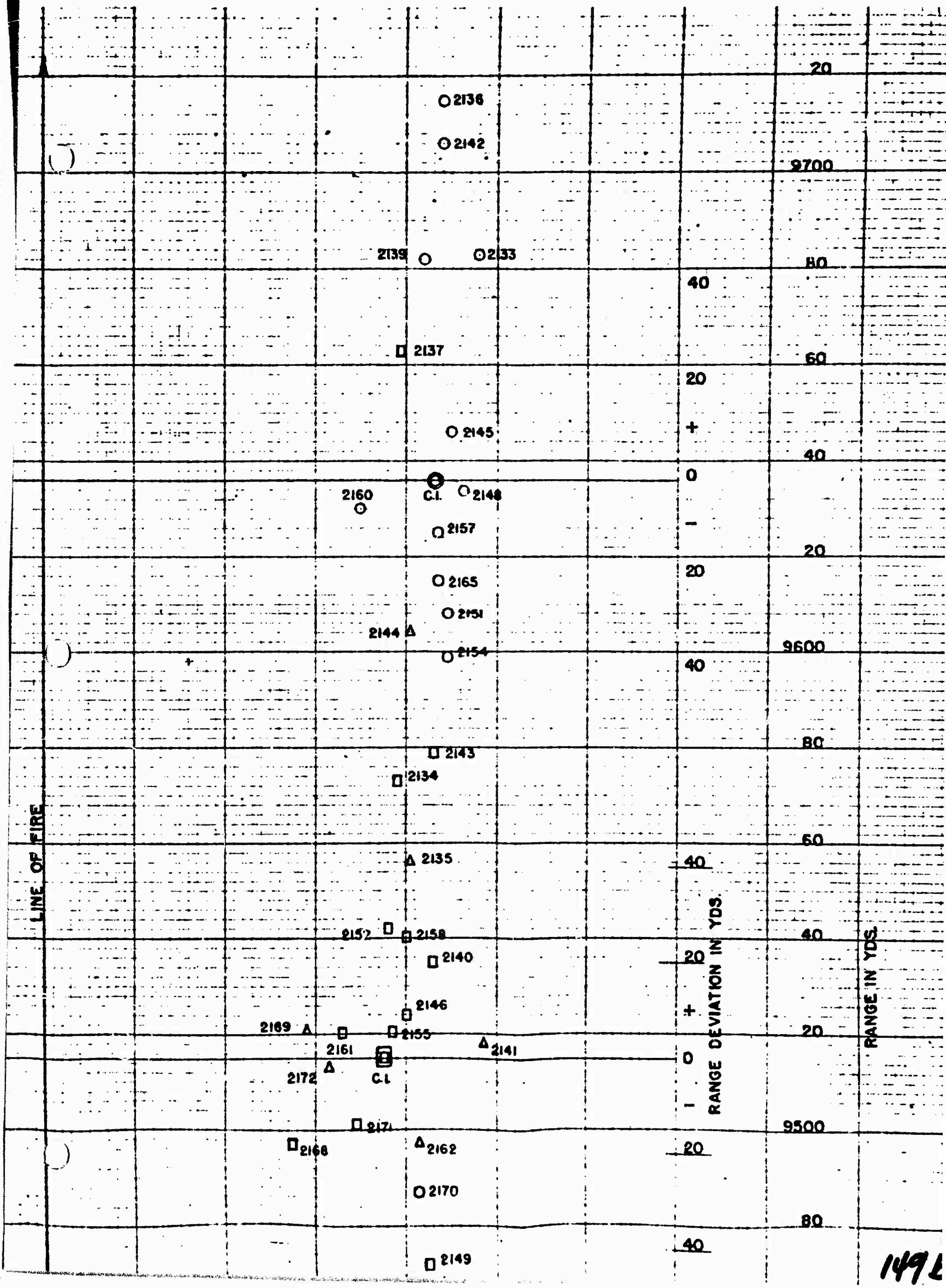
C. M. WESSON,
Col., Ord.Dept.,
Commanding.

for

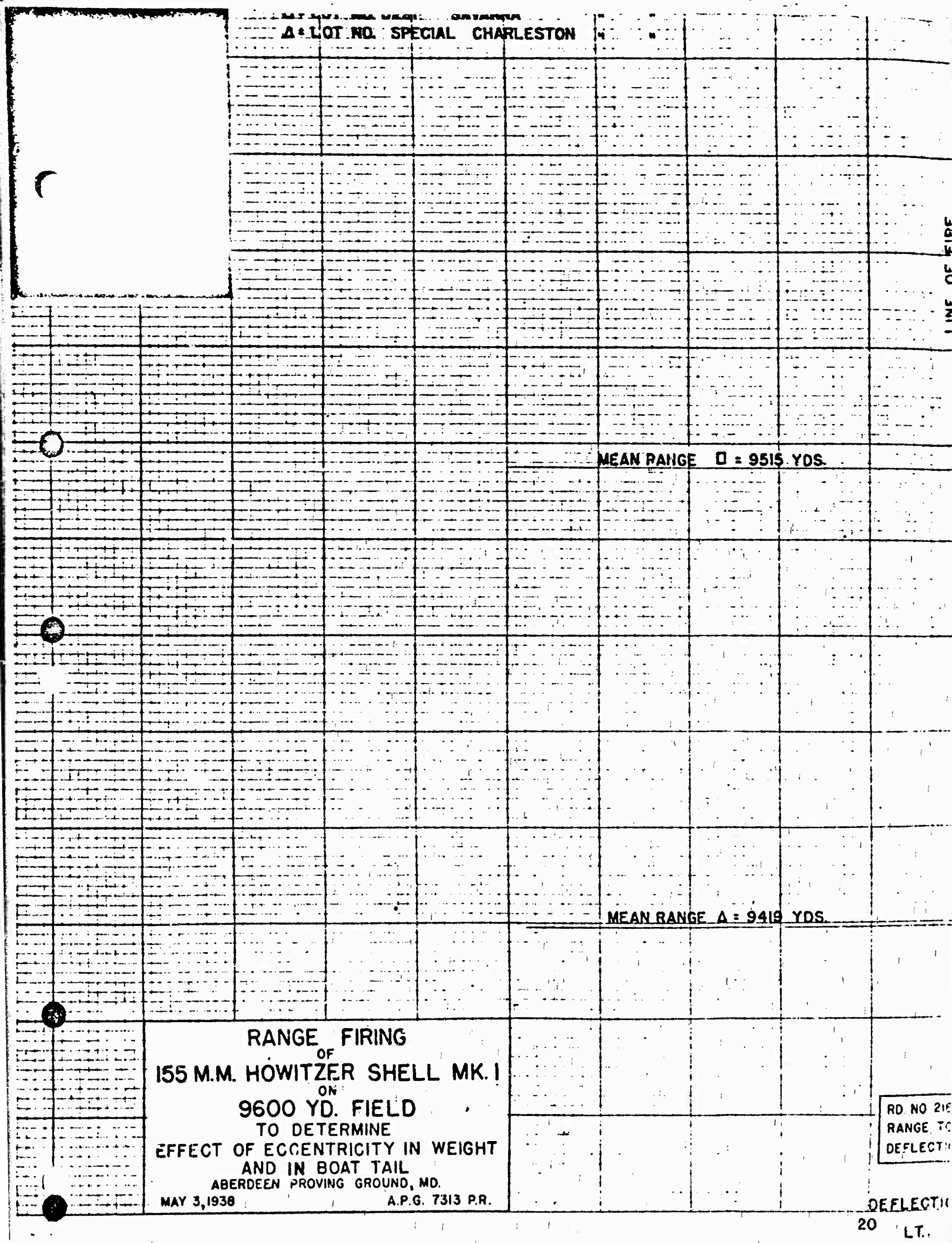
APPENDIX C

RANGE FIRING OF 155MM. HOWITZER H.E. SHELL MK.I
ON 9600 YD. FIELD TO DETERMINE EFFECT
OF ECCENTRICITY IN WEIGHT AND IN BOATTAIL

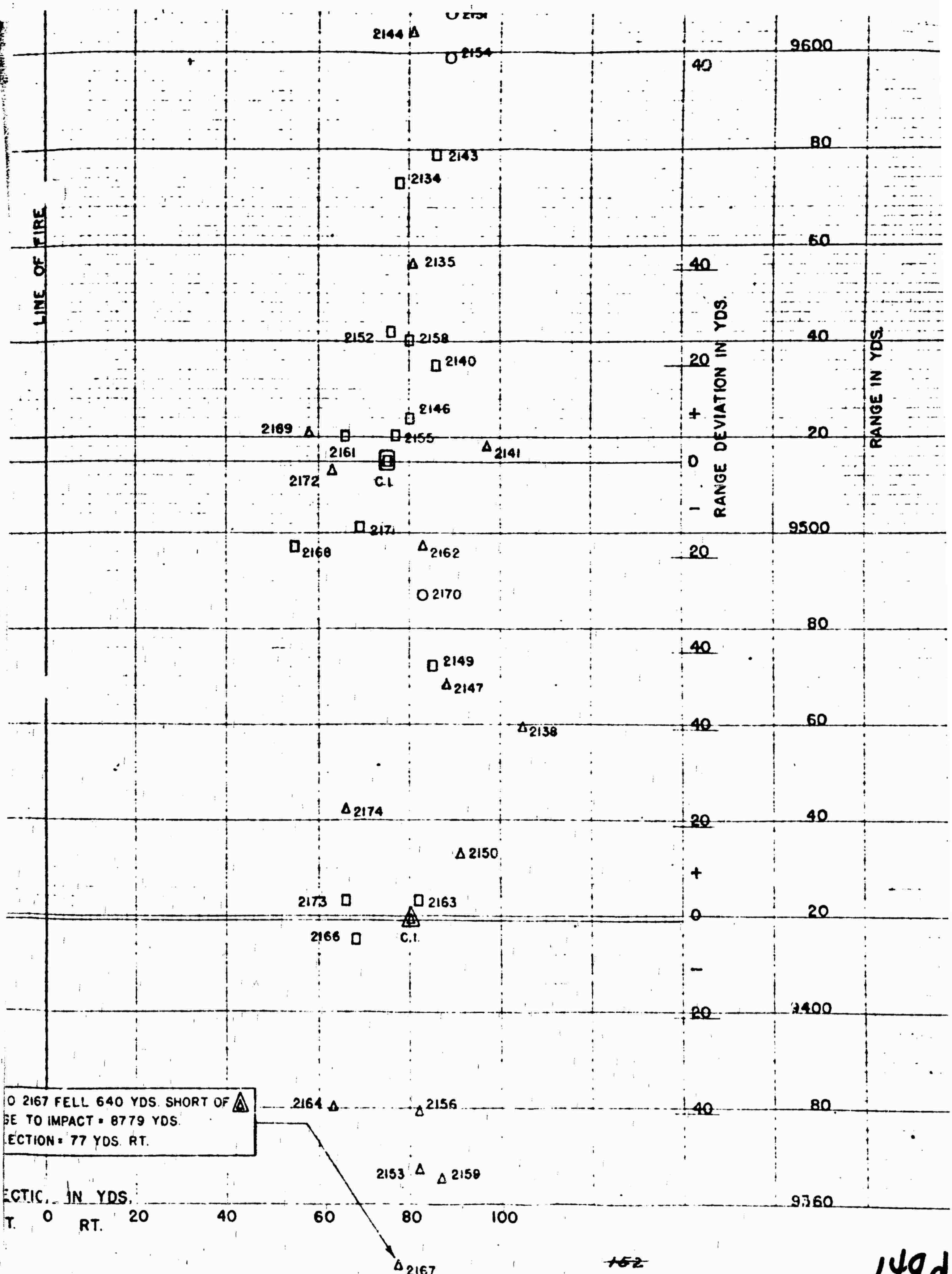

MEAN RANGE ○ = 9636 YDS.
SYMBOLS
○ = LOT NO. 7884-1 SHELL MK.I
□ = LOT NO. 5291- SAVANNA " "
Δ = LOT NO. SPECIAL CHARLESTON " "
LINE OF FIRE
MEAN RANGE □ = 9515 YDS.


LINE OF FIRE
RANGE DEVIATION IN YDS.
RANGE IN YDS.
20
9700
80
60
40
20
9600
80
60
40
20
9500
80
40
20
+
0
−
20
40
40
20
+
0
−
20
40
2136
2142
2139
2133
2137
2145
C.I.
2148
2160
2157
2165
2151
2144
2154
2143
2134
2135
2152
2158
2140
2146
2169
2161
2155
2141
2172
C.I.
2171
2168
2162
2170
2149

149L


Δ = LOT NO. SPECIAL CHARLESTON
LINE OF FIRE
MEAN RANGE □ = 9515 YDS.
MEAN RANGE Δ = 9419 YDS.
RANGE FIRING
OF
155 M.M. HOWITZER SHELL MK. I
ON
9600 YD. FIELD
TO DETERMINE
EFFECT OF ECCENTRICITY IN WEIGHT
AND IN BOAT TAIL
ABERDEEN PROVING GROUND, MD.
MAY 3, 1938
A.P.G. 7313 P.R.
DEFLECTI
20
LT.

149c



LINE OF FIRE
RANGE DEVIATION IN YDS.
RANGE IN YDS.
9600
80
60
40
20
9500
80
60
40
20
9400
80
9360
C.I.
C.I.
0 2167 FELL 640 YDS. SHORT OF
GE TO IMPACT = 8779 YDS.
LECTION = 77 YDS. RT.
ECTIC.. IN YDS.
T. 0 RT. 20 40 60 80 100
2144
2154
2143
2134
2135
2152
2158
2140
2146
2169
2161
2155
2141
2172
2171
2168
2162
2170
2149
2147
2138
2174
2150
2173
2163
2166
2164
2156
2153
2159
2167

~~152~~

149d

APPENDIX D

METEOROLOGICAL DATA

ORDNANCE METEOROLOGICAL SERVICE

Aberdeen Proving Ground, Md.

To Ballistic Section:

Densities aloft as determined by temperature flight with C-7 Altimeter during the period 10:20 A.M. to 12:16 P.M., May 3, 1938.

Altitude yds.	Temperature Fahrenheit True	Temperature Fahrenheit Mean	Density
0	71.0	71.0	0.992
170	69.0	70.0	0.992
340	65.0	68.3	0.994
509	63.0	67.0	0.993
681	61.5	65.9	0.993
849	60.5	65.0	0.993
1021	59.5	64.2	0.993
1193	58.5	63.5	0.991
1361	57.0	62.8	0.990
1534	56.5	62.2	0.990
1702	54.5	61.5	0.988

Surface Data

Time	Temperature	Pressure	Density
10:20 A.M.	73°	30.05	0.993
12:16 P.M.	75°	30.00	0.986

ORDNANCE METEOROLOGICAL SERVICE

Aberdeen Proving Ground, Md.

To Ballistic Section:

Densities aloft as determined by temperature flight with C-5 Altimeter during the period 2:02 P.M. to 2:35 P.M., May 3, 1938.

Altitude yds.	Temperature Fahrenheit True	Mean	Density
0	76.6	76.6	0.975
172	73.0	74.8	0.977
344	70.7	73.4	0.979
515	68.5	72.3	0.980
688	65.8	70.9	0.980
858	62.6	69.6	0.980
1028	60.4	68.2	0.981
1197	58.1	66.9	0.981
1371	57.2	65.8	0.981

Surface Data

Time	Temperature	Pressure	Density
2:02 P.M.	76.1°	29.93	.983
2:35 P.M.	77.9°	29.91	.980

Ascension No. 1 Date: May 3, 1938. Time 9:38 A.M.

No. Theodolites Used: 2 Station Used: O-Camera C-Tower

Azimuth Line of Fire = 36° 27'

Time Observed Min.	Altitude yds.	Wx	Wz
0	0	+1.0	+ 4.9
1	250	+5.5	+ 4.2
2	480	+5.6	+ 5.8
3	660	+5.7	+ 6.0
4	860	+4.6	+ 3.8
5	1080	+4.0	- 3.2
6	1270	+5.2	-12.0
7	1480	+8.1	-17.7
8	1670	+10.1	-20.7
9	1840	+10.0	-21.3
10	2010	+9.9	-20.5

Visibility: Good Sun: Bright Temperature: 72° F.

Pressure: 30.04

Disappearance: Abandoned. Relative Humidity: 43%

Surface Wind: Direction (To) 115 Velocity: 9 m.p.h.

Ascension No.: 2 Date: May 3, 1938 Time: 10:42 A.M.

No. Theodolities Used: 2 Station Used: O-Camera C-Tower

Azimuth Line of Fire: 36° 27'

Time Observed	Altitude yds.	Wx	Wz
0	0	+ 4.8	+ 6.4
1	260	+ 4.0	+ 4.8
2	450	+ 3.5	+ 5.0
3	610	+ 2.8	+ 5.8
4	810	+ 2.6	+ 4.6
5	1050	+ 2.0	+ 1.0
6	1270	+ 3.0	- 8.0
7	1490	+ 8.6	-16.8
8	1690	+12.5	-19.8
9	1870	+12.8	-21.4
10	2080	+13.0	-22.2

Visibility: Good Sun: Bright Temperature: 73° F.

Pressure: 30.04 in.

Disappearance: Abandoned Relative Humidity: 46%

Surface Wind: Direction: (To) 90 Velocity: 8 m.p.h.

Ascension No.: 4 Date: May 3, 1938 Time: 12:04 P.M.

No. Theodolities Used: 1 Station Used: O-Camera

Azimuth Line of Fire: 36° 27'

Time Observed	Altitude yds.	Wx	Wz
0	0	- 2.6	+ 4.3
1	240	- 3.2	+ 6.3
2	460	- 2.5	+ 8.4
3	680	+ 0.4	+ 5.2
4	890	+ 0.9	+ 3.6
5	1100	+ 4.5	- 9.2
6	1300	+ 8.1	-17.6
7	1500	+10.0	-17.5
8	1700	+12.1	-22.4
9	1900	+11.3	-23.3
10	2100	+11.2	-23.4

Visibility: Good Sun: Bright Temperature: 74° F

Pressure: 30.00 in.

Disappearance: Abandoned Relative Humidity: 43%

Surface Wind: Direction: (To) 150 Velocity: 5 m.p.h.

Ascension No.: 6 Date: May 3, 1938 Time: 1:15 P.M.

No. Theodolites Used: 2 Station Used: O-Camera C-Tower

Azimuth Line of Fire: 36° 27'

Time Observed	Altitude yds.	Wx	Wz
0	0	-1.2	+ 4.9
1	290	+ 1.4	+ 4.8
2	470	- 6.2	+ 5.6
3	720	- 7.0	+ 4.8
4	960	- 6.8	+ 3.9
5	1160	-11.2	- 0.4
6	1390	-17.8	- 3.2

Visibility: Good Sun: Bright Temperature: 76° F

Pressure: 29.98 in.

Disappearance: Burst in Air Relative Humidity: 43%

Surface Wind: Direction: (To) 140 Velocity: 5 m.p.h.

Ascension No.: 7 Date: May 3, 1938 Time: 2:08 P.M.

No. Theodolites Used: 2 Station Used: O-Camera C-Tower

Azimuth Line of Fire 36° 27'

Time Observed	Altitude yds.	Wx	Wz
0	0	-1.2	+ 4.9
1	300	-6.0	+ 3.3
2	660	-8.4	+ 2.6
3	1080	-3.6	- 1.1
4	1430	+4.2	-11.7
5	1830	+6.0	-21.8
6	2190	+5.2	-27.8
7	2590	+2.7	-32.6
8	3030	-1.6	-31.2
9	3470	-2.3	-32.1
10	3850	+2.7	-40.0
11	4220	+3.6	-44.2
12	4580	0.0	-44.8
13	4930	-1.2	-45.0
14	5230	+0.6	-45.0
15	5580	+2.0	-45.0

Visibility: Good Sun: Bright Temperature: 77° F

Pressure: 29.98 in.

Relative Humidity: 40%

Surface Wind: Direction: (To) 140 Velocity 5 m.p.h.

APPENDIX E

MEASUREMENT OF
CRITICAL DIMENSIONS
OF STANDARD SHELL
AND OF ECCENTRIC
CHARLESTON AND
SAVANNA SHELL

Measurements made on twelve 155 mm Howitzer Shell, Type Mark I, Savanna Ordnance Depot.
O.P. 4703.

Shell Number	Lot Number	Distance, base to band	Distance, rear of band to boat tail Max.	Min.	Dia., shell body at rear of band Max.	Min.	Dia., shell body between band and bourrelet Max.	Min.	Diameter of bourrelet Max.	Min.
1	7884-1	3.22	.52	.47	6.055	6.053	6.043	6.043	6.079	6.079
2	7884-1	3.23	.58	.55	6.057	6.054	6.049	6.046	6.081	6.080
3	7884-1	3.20	.67	.60	6.042	6.036	6.036	6.035	6.077	6.077
4	7884-1	3.22	.97	.95	6.026	6.023	6.037	6.036	6.079	6.079
5	7884-1	3.22	.54	.49	6.023	6.021	6.039	6.038	6.079	6.079
6	7884-1	3.20	.30	.28	6.046	6.042	6.049	6.046	6.080	6.080
7	7884-1	3.24	.55	.53	6.029	6.027	6.034	6.034	6.080	6.080
8	7884-1	3.24	.61	.54	6.044	6.041	6.049	6.049	6.082	6.080
9	7884-1	3.20	.52	.47	6.034	6.031	6.028	6.028	6.078	6.077
10	7884-1	3.22	.62	.60	6.038	6.034	6.042	6.040	6.079	6.078
11	7884-1	3.23	.62	.58	6.057	6.056	6.054	6.052	6.079	6.079
12	7884-1	3.25	.45	.40	6.047	6.045	6.037	6.034	6.084	6.080

Gauge Section,
Aberdeen Proving Ground,
Maryland.

Arthur E. Jewell,
Principal Proof Technician.

Measurements made on fifteen 155 mm Howitzer Shell, Type Mark I. Eccentric with respect to weight distribution and center of gravity. Received from Charleston Ordnance Depot.
O.P. 4703.

Shell Number	*Amount of weight to balance (ounces)	Distance, base to band	Distance, rear of band to boat tail Max.	Distance, rear of band to boat tail Min.	Dia., shell body at rear of band Max.	Dia., shell body at rear of band Min.	Dia., shell body between band and bourrelet Max.	Dia., shell body between band and bourrelet Min.	Diameter of bourrelet Max.	Diameter of bourrelet Min.
1C	62	3.21	.90	.77	6.055	6.055	6.041	6.041	6.079	6.076
2C	58	3.16	.94	.80	6.029	6.022	6.016	6.013	6.073	6.073
3C	34	3.15	.96	.87	6.006	6.005	6.036	6.033	6.081	6.080
4C	34	3.19	.68	.65	6.043	6.038	6.047	6.047	6.073	6.072
5C	28	3.15	.85	.75	6.059	6.056	6.046	6.045	6.073	6.042
6C	28	3.19	.85	.80	6.039	6.037	6.043	6.043	6.081	6.081
7C	27	3.23	1.22	1.15	6.012	6.010	6.087	6.085	6.078	6.036
8C	27	3.17	1.35	.75	6.026	6.019	6.024	6.024	6.078	6.075
9C	27	3.19	.87	.85	6.059	6.055	6.033	6.031	6.072	6.061
10C	26	3.16	.90	.77	6.061	6.060	6.034	6.034	6.081	6.077
11C	26	3.17	.83	.83	6.043	6.042	6.036	6.036	6.078	6.073
12C	26	3.17	1.05	.61	6.010	6.007	6.010	6.093	6.076	6.065
13C	26	3.16	.94	.87	6.029	6.026	6.023	6.020	6.076	6.075
14C	26	3.17	.82	.80	6.046	6.046	6.039	6.039	6.079	6.079
15C	25	3.15	.77	.73	6.028	6.028	6.030	6.030	6.079	6.078

* Determined at Charleston Ordnance Depot.

Gauge Section,
Aberdeen Proving Ground,
Maryland.
J

Arthur E. Jewell,
Principal Proof Technician.

Measurements made on fifteen 155 mm Howitzer Shell, Type Mark I, with eccentric boat tails.
Received from Savanna Ordnance Depot. O.P. 4703.

Shell Number	Lot Number	*Eccentricity of boat tail	Distance base to band	Distance, rear of band to boat tail Max.	Distance, rear of band to boat tail Min.	Dia., shell body at rear of band Max.	Dia., shell body at rear of band Min.	Dia., shell body between band and bourrelet Max.	Dia., shell body between band and bourrelet Min.	Diameter of bourrelet Max.	Diameter of bourrelet Min.
1S	5291-1	.045	3.17	.82	.78	6.018	6.018	6.028	6.025	6.078	6.076
2S	5291-1	.040	3.17	.83	.78	6.018	6.016	6.036	6.034	6.082	6.079
3S	5291-1	.047	3.15	.77	.72	6.025	6.023	6.028	6.027	6.083	6.079
4S	5291-1	.044	3.16	.93	.73	6.009	6.008	6.043	6.021	6.082	6.080
5S	5291-4	.030	3.19	.87	.75	5.994	5.978	6.022	6.028	6.077	6.077
6S	5291-4	.030	3.18	.90	.87	5.989	5.958	6.025	6.025	6.081	6.080
7S	5291-4	.043	3.16	.95	.78	5.997	5.970	6.038	6.034	6.082	6.069
8S	5291-4	.015	3.15	.87	.65	6.021	6.021	6.049	6.022	6.081	6.076
9S	5291-4	.030	3.16	1.02	.65	6.041	6.040	6.042	6.040	6.084	6.080
10S	5291-5	.035	3.16	.78	.73	6.017	6.015	6.037	6.028	6.082	6.082
11S	5291-26	.033	3.16	1.02	.75	6.000	6.000	6.017	6.015	6.079	6.068
12S	5291-27	.020	3.16	1.05	.93	5.988	5.986	5.986	5.983	6.079	6.078
13S	5291-27	.023	3.16	1.01	.88	5.978	5.977	6.018	6.004	6.079	6.044
14S	5291-27	.020	3.16	1.15	1.03	6.006	6.006	6.031	6.024	6.079	6.079
15S	5291-27	.018	3.15	.85	.76	5.998	5.993	6.022	6.016	6.081	6.080

* Measured at Savanna Ordnance Depot.

Gauge Section,
Aberdeen Proving Ground,
Maryland.
J

Arthur E. Jewell,
Principal Proof Technician.